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AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

LARGE CORE DRILLS AID CONSTRUCTION AT CHICKAMAUGA DAM

By James S. Lewis, Jr.,[1] Assoc. M. Am. Soc. C. E.

Synopsis

The use of large core drills for the purpose of investigating foundations, and for supplementing and verifying the information obtained with small borings, is becoming increasingly common in conjunction with the construction of heavy hydraulic structures. However, the use of these large drills as a direct means of furthering the ends of construction is somewhat unusual, and this paper contains a description of the methods that were successfully used, under difficult working conditions, to obtain a foundation for the upper guard wall of the Chickamauga (Tenn.) Project navigation lock. The equipment and its operation are described in detail, and cost and progress information is included.

Introduction

The preliminary borings, made for the purpose of investigating the site of Chickamauga Dam, near Chattanooga, Tenn., indicated that unusual foundation difficulties would probably be encountered in part of the area covered by the structure. The subsequent excavation in the lock area revealed, however, that with the exception of the up-stream end of the river guard wall, the lock walls could be rested upon ledge rock without unusual difficulty. Under the upper end of this wall, the rock was covered by a heavy layer of over-burden through which it was both unsafe and uneconomical to attempt to excavate, and it was decided to support the piers carrying the wall upon reinforced concrete columns extending through the over-burden and weathered limestone to sound rock.

The guard wall (see Fig. 1) consists essentially of a number of cantilever sections that are supported by piers. Each pier supports a section 42 ft long, 20 ft deep, and 8 ft thick that spans half the distance to the adjacent piers. From its junction with the lock wall, the guard wall is 264 ft long. Each pier is 14 ft by 40 ft in plan at the base, and the side away from the navigation

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by October 15, 1939.

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channel is battered so that the top width of the pier equals the thickness of the cantilever wall. This description does not apply to the nose pier, which is rounded on the up-stream side in a conventional manner.

Fig. 2 shows the general plan of the lock, and this structure, with the exception of Piers 27 through 32, was founded directly upon the Chickamauga limestone that composes the massive geologic formation of the region. The information that had been obtained from the early borings caused considerable suspicion with regard to the area to be covered by the guard wall, and additional borings were made when the lock coffer-dam was unwatered. Holes were drilled at the four corners of each of the proposed pier locations, and it was learned that the apprehension that had been felt was fully justified. The over-burden in the area of the piers was found to be more than 60 ft thick in places and was composed of soft clay containing numerous boulders of varying size. Heavy flows of water were not generally encountered in the over-burden, but it was not entirely impervious, and the saturated condition of the material resulted in a lack of stability that would have threatened the safety of any open excavation that might have been attempted.

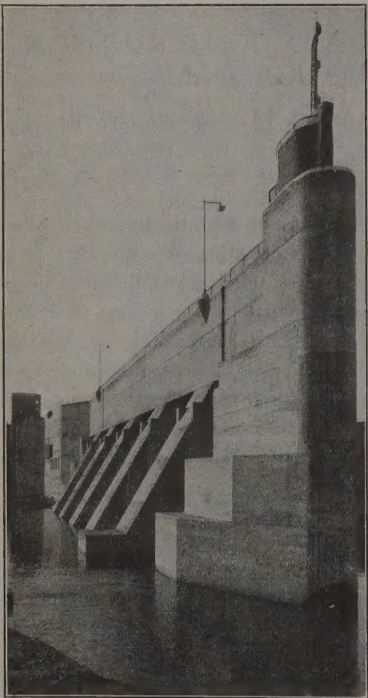

Fig. 1.—Completed Guard Wall

The surface of the underlying ledge rock was of the extremely irregular contour that is typical of weathered limestone and was characterized by frequent pinnacles, pot-holes, and deep, clay-filled crevices. The ledge rock itself was characterized by the existence of extremely sharp folds and numerous faults, and contained well-defined layers of shale and bentonite. The latter was comparatively soft, having little bearing strength when unrestrained, and was frequently found in contact with the shale. Numerous seams and weathered joints and fissures existed to afford passage for the ground-water.

In consideration of the uncertainties and difficulties attendant upon securing a satisfactory foundation of a conventional type under the conditions that existed, reinforced concrete columns, extending through the over-burden and weathered limestone to bear upon sound rock, were chosen as the most eco-

nomical and satisfactory method of supporting the piers. Sound rock was usually found below a generally prevalent stratum of shale and bentonite that formed a very irregular bed below which the ledge rock was comparatively free from large seams and openings. However, the sharp folding in some areas

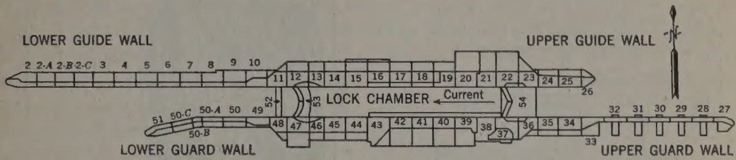

Fig. 2.—Schematic Plan; Chickamauga Lock

made it necessary that the columns pass through layers of sound rock that overlay weathered material, above which it was not considered wise to stop. Each pier, with the exception of the one that formed the nose of the guide wall, was supported by six columns as shown in Fig. 3. All of the columns were similar in design, having a required minimum diameter of 36 in., and longitudinal reinforcing bars with circular hoops.

Drilling Equipment

Of the various schemes considered, large core drills were selected as offering the greatest promise for satisfactorily sinking the holes for the columns, and the area was prepared in advance to facilitate the operation of the equipment. The surface of the over-burden was so soft and uneven that working conditions were very difficult and, before the drilling was started, a rectangular slab of reinforced concrete, of the size of the pier and 3 ft thick, was poured over the location of each pier. This slab was intended to afford a firm support for the drills as well as better footing for the operators and to prevent the loss of small tools and parts that might otherwise have become covered by the soft mud. Four large drills were used during the course of the work, three of which were originally designed for 36-in. holes, and one of which was designed for drilling 72-in. holes. In order to drill the larger holes required for obtaining the minimum diameter at the bottom of the columns, it was necessary to install 40-hp drive motors to replace the 15-hp units with which two of the smaller drills were equipped. The third 36-in. drill was driven by a gasoline motor. It was found that neither type of driving power possessed any decided advantages under ordinary circumstances. The 36-in. drills were mounted on skids and weighed approximately 3 tons each when fully equipped. They could be moved with the hoist on the drill or with a crane or tractor.

The fourth machine was designed for drilling 72-in. holes and was of much larger size and heavier construction throughout. Instead of being mounted on skids, it was equipped with six wheels designed to roll on rails 8 ft apart, and it was necessary to place track over the location of any hole to be drilled. The spindle was in the center of the machine instead of in the customary position at the end and, as a result, the hole was entirely covered by the drill when it was in the operating position. To remove the tools and core, it was necessary to

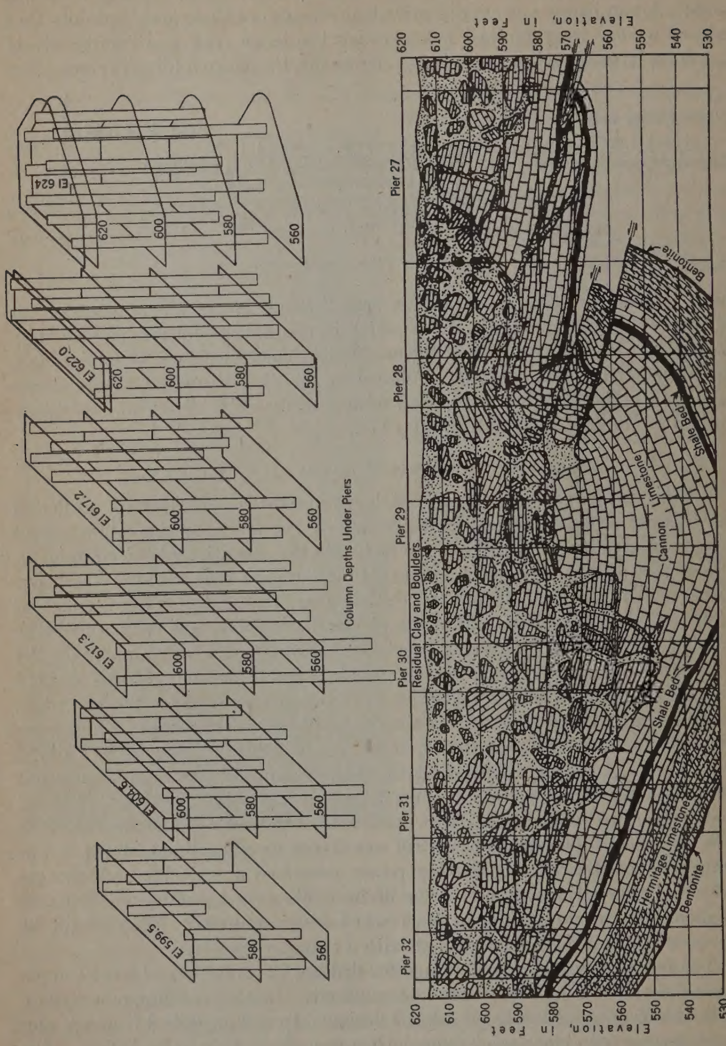

FIG. 3—GENERALIZED GEOLOGIC SECTION UNDER UPPER GUARD WALL

roll the drill back upon the track until the hole was clear, when the derrick mounted upon the end of the unit was used for hoisting from the hole. The weight of the drill rods with which this machine was equipped was so great that they were difficult to handle and the progress suffered as a result. As compared with the 36-in. machines, the cutting rate of the large drill was probably greater under similar conditions; but, the time lost in handling the heavy drill rods and in moving and setting up the large machine more than offset this advantage. Under different conditions, where the surface was fairly even and where the holes were placed on a line and spaced closely enough to justify laying a continuous track, the large drill could probably be operated to greater advantage. The weight of the 72-in. machine erected was approximately 9 tons and, to

FIG. 4.—GENERAL VIEW OF OPERATIONS

avoid serious loss of time, a crane was always used for moving it to new locations. The various sizes of drills that were used may be seen in Fig. 4.

The drills were of the shot type and consisted essentially of a vertical hollow spindle rotated by the power unit through a system of gears, or gears with chains and sprockets. The spindle was connected by hollow drill rods to the bit below. As the bit rotated, shot was transported in a small stream of water through the hollow spindle and rods to the cutting edge. The design of this bit is a matter of importance if the maximum cutting efficiency is to be obtained, and the type found to be most satisfactory is shown in Fig. 5. The liner is desirable for drilling in deep holes, as it prevents the bottom from wearing to a sharp edge under which shot will not remain. Furthermore, as the holes become deeper and the weight of the tools increases, the additional width of the cutting edge prevents the shot from being forced aside, where they are in-

effective. The core is cut smaller, giving it less tendency to bind and block in the barrel, and the offset formed by the top of the liner occasionally grips pieces of loose core which are brought to the surface with the bit. The liner is not needed when the holes are shallow. The shot-spreader plate deflects the shot outward so that they fall under the cutting edge rather than on top of the core projecting upward in the bit.

It was required that the columns should have a minimum diameter of 36 in. and, to insure that this size would be obtained at the bottom, it was necessary to begin the holes much larger at the top. The bits varied in outside diameter from 49 in. to 37 in., decreasing in 2-in. increments, and the casing varied from 48 in. to 36 in. inside diameter, also decreasing in 2-in. increments. As shown in Fig. 5, the thickness of the plate from which the bits were rolled varied according to the diameter. The casing was rolled from $\frac{1}{4}$-in. plate and was purchased in sections 10 and 15 ft long, with a few 5-ft sections. By standardizing upon the sizes indicated, the bit in use always had a clearance of $\frac{1}{2}$ in. when centered in the smallest casing through which it was necessary for it to pass. A clearance of $\frac{1}{4}$ in. was found to be insufficient, as it was impractical to keep either bit or casing perfectly round, and trouble frequently resulted when the bit could not be passed through the casing.

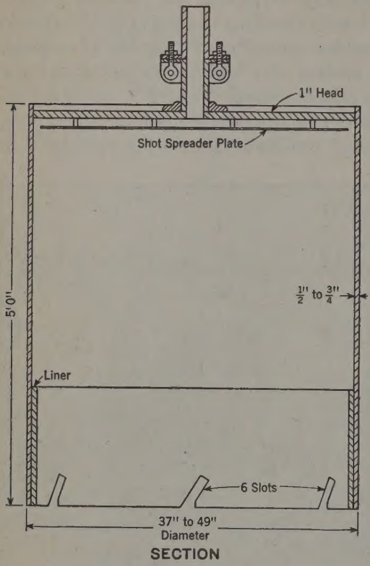

Fig. 5.—Typical Shot Bit for Large Holes

Operation of Drills

When a bit full of core had been cut, or conditions in the hole made it desirable to stop drilling, the tools were removed so that a man could be lowered to the bottom. Ordinarily, to cut more than 3 ft without removing the bit was considered fair; and a run of more than 4 ft was unusual. After removing the tools, a jackhammer hole was drilled in the center of the core and a pin with hoisting ring was wedged tightly in the small hole. A light charge of dynamite was then placed in the groove on one side of the core and the man was removed from the hole; a strain was taken upon a hoisting line fastened to the pin wedged in the core and the dynamite was discharged. The core could then frequently be raised to the surface with the hoist. If the core did not break at the bottom, so that all of it could be removed in one piece, it was necessary

to fire another charge of dynamite beside the remaining core. It was found that a light charge that just sufficed to crack the core gave better results than a charge heavy enough to shatter the rock and make its removal in fragments necessary. Since it was necessary to handle the drills, casing, bits, and cores at frequent intervals, a crawler crane was kept in constant attendance upon the drills throughout most of the operation.

In an effort to improve the progress and to reduce the trouble resulting from caving of the walls of the holes, a bit 11 ft long and 42 in. in diameter, with a removable head, was made. It was thought that this bit could be rotated down in the usual manner until the barrel was filled. The head and drill rods would then be removed and, while the bit supported the side walls against caving, the muck or core would be removed from the inside. After emptying the bit, it was planned to resume drilling without removing it from the hole and to follow its progress closely with casing to insure against caving from above. In this manner, it was hoped to prevent the caving which frequently occurred when the bit was removed after a cut had been made. It would have been necessary to abandon a bit, with head removed, in the bottom of each completed hole, as it would have been impossible to remove it through the casing above. However, the scheme proved to be unsatisfactory for several reasons. The head could be removed only with the greatest of difficulty and delay. The increased friction between the long bit and the material through which it was passing over-loaded the drill motor. If sufficient progress had been made, it probably would have been found that the setting of the casing caused additional difficulty. It is possible that this idea might have been modified and improved to give the desired results, but the pressing schedule precluded further experimentation and the more certain methods were again adopted.

When this work was begun, it was thought that, by following the progress of the mucking and drilling closely with the casing, difficulties resulting from caving of the walls of the holes might be avoided. However, soon after operations were begun, it became evident that the persistent caving of the walls, where the saturated and unstable over-burden was penetrated, was delaying progress seriously, and the heavy flows of water that were encountered in the rock made a complete cessation of drilling necessary at times. The nature of the material composing the over-burden made it impracticable to lower the water-table in the area by pumping from deep holes drilled for that purpose, and it was necessary to take measures to exclude the water in the immediate vicinity of each hole.

Advance Grouting to Exclude Water

Advance grouting in stages was chosen as the system offering the greatest promise of consolidating the over-burden and sealing the openings in the rock satisfactorily, and 5.5-in. shot core holes were drilled and grouted in the center of each column location before the large holes were started. These small holes were grouted at the completion of each stage of drilling so that every stage except the last was subjected to repeated injections of grout under pressure. The depth of the various stages was not determined in advance but depended

upon the nature of the material encountered. When it was necessary to penetrate a considerable thickness of over-burden to reach rock, the grouting was accomplished in several stages. The first would be drilled down probably 10 ft and, if the material penetrated was soft, loose, caving, or otherwise unstable, the drill tools were withdrawn, and a 1-in. pipe was inserted to the bottom of the hole. The hole was then grouted through the circulating arrangement shown in Fig. 6. By grouting through the pipe extending to the bottom, the grout in the hole was kept alive, or liquid, at all times, and material that caved

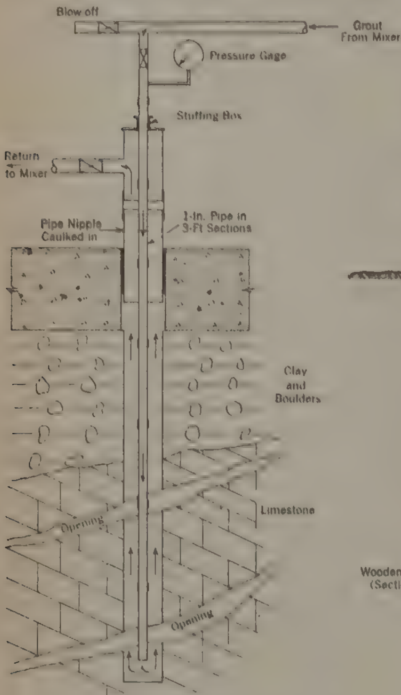

Fig. 6.—Full Circulating Arrangement for Stage Grouting of Over-Burden and Rock

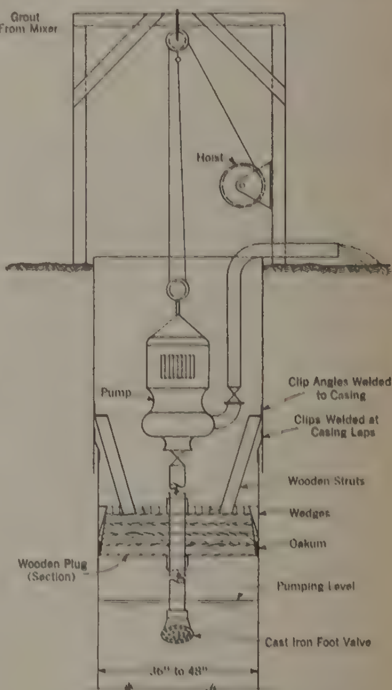

Fig. 7.—Installation of Wooden Plug for Grouting through Large Holes

from the sides was prevented from permanently blocking off any part of the hole. The pressure in the hole was regulated by the valve in the return line, and a stuffing box in the top of each header allowed the 1-in. pipe to be kept free by raising and lowering it at frequent intervals while grouting. The pipe was composed of 3-ft sections so that, when sticking was threatened, it could be raised and a section removed with only a momentary interruption of the flow. In this manner, the pipe was removed slowly, 3 ft at a time, until the hole refused to take more grout. An effort was made to grout to refusal at a pressure

which varied between 25 and 50 lb per sq in. in the over-burden and as much as 100 lb per sq in. in rock. When surface leaks occurred, the rate of pumping was reduced; empty sacks were packed into any large openings and covered with sand bags. This usually retarded the flow sufficiently to allow the grout to harden and plug the leak. When the first stage was grouted, drilling was resumed through the same hole and extended to such depth as was determined to be suitable for the second stage. The advance grouting was thus carried down in each column location until the cores recovered from the pilot holes disclosed that sound rock had been reached.

In addition to the value of this preliminary work for consolidating the over-burden and plugging the openings in the rock, the pilot holes afforded a reliable means of determining in advance the probable depth to which each large hole would have to be drilled.

For the grouting of the over-burden and rock, a very thick mixture of cement and water containing 3% of calcium chloride by weight of the cement was used. The calcium chloride was added to accelerate the setting action of the cement and thus prevent the grout from traveling to distant areas where it was not needed. It follows that less cement was required. The mixture was handled by air-driven, reciprocating pumps, and the first few batches were ordinarily mixed with a water-cement ratio (by weight) of 0.66. If the hole accepted the grout freely, the ratio was reduced to 0.40 and possibly later to 0.33 if no resistance was offered. Steady pumping was continued until refusal occurred at the desired pressure, or until surface leaks or other contingencies made it desirable to change to an intermittent flow, or to cease entirely. The very thick grout was deemed superior to the more fluid mixtures because of the greater tendency of the latter to open and erode passages through the over-burden. Furthermore, the thicker grout sets faster and probably shrinks less.

The advance grouting, however, did not serve as an infallible means of eliminating the hazard imposed by the ground-water, as there were instances when water later broke into the large holes through over-burden or rock despite all previous efforts to exclude it. The over-burden was definitely consolidated and stabilized to a marked degree, and its density and resistance to percolation were increased. When the grouted material was removed, it was always found to contain a quantity of hardened grout in large pieces, and the clay immediately surrounding the hole was well diffused with unhardened cement. This consolidation of the material surrounding the holes prevented the seeps that often preceded flows of large volume. Following the advance grouting, it was generally true that less trouble from water was encountered in the over-burden than in the rock. This is probably best explained by the fact that in the rock the water followed well-defined channels, whereas in the over-burden it simply caused a generally saturated condition. Thus, when the water followed a narrow vertical fissure or a channel that passed just within the boundary of the large hole, the small hole for advance grouting might fail to intercept the water bearing opening. When the large drill cut into this ungrouted source, of course, the hole was flooded quickly. It was then necessary to regrout the hole by means of a wooden plug which will be described subsequently. Judged as a

whole, the advance grouting was quite successful and improved conditions to an extent that made the work possible with the methods used.

Drilling and Mucking

Following the completion of the grouting of any location, a large drill was placed upon the concrete working slab and the hole was started with a 49-in. bit. To avoid confusion, bits were designated by the outside diameter, and casing was designated by the inside diameter. Although all bits were 5 ft long, it was only with rare good luck that sufficient core could be cut to fill the barrel. Blocking of the core, caving of the wall of the hole, or other trouble, usually made it necessary to remove the tools before a full barrel had been cut. A sound, homogeneous rock would no doubt make the use of a longer bit desirable and would result in improved progress by reducing the frequency with which it was necessary to remove the core. The hole was drilled through the concrete slab and into the over-burden with the 49-in. bit until the walls of the hole appeared to be in danger of caving. This distance was ordinarily 10 ft or less. In the boulder-strewn over-burden, the bit would cut a clear opening of the desired size until the 5-ft length was full, or until blocking or binding made it necessary to stop. The bit was then removed and the muck was excavated by hand and brought to the surface by the air hoist on the drill. The removal of the muck ordinarily required more time than the actual drilling as the limited space allowed only one man to work in the hole at any one time. If, after removing the muck to the depth to which the bit had cut, an examination disclosed that the walls were in danger of caving, the exposed part was cased and the size of the bit was reduced for the next cut.

Walls that were apparently safe against caving would sometimes fall in when drilling was resumed and swirling drill water eroded and saturated the standing material. As it was necessary to pump the water from the hole each time that core or muck was to be removed, any seepage through the over-burden tended to increase when the head opposing it was removed. Two-stage, air-driven, centrifugal pumps were used for unwatering the holes as their light weight and compact construction gave them the advantage over other types. The leakage tended to increase as the passages through which the water was flowing were eroded to a larger size, and sudden blow-ins of sufficient volume to interrupt the drilling seriously were not infrequent during the early stages of the program.

In an effort to prevent the blow-ins, the space between the casing and the wall of the hole was well packed with dry cement or dry mortar as each piece of casing was set. The size of the space to be filled varied from practically nothing to several inches, and the type of material that was to be used for packing was determined by the size of the opening. Quick-setting cement was used when the volume of leakage indicated its desirability. It was found that the tendency of small leaks to increase to damaging proportions was eliminated if this space behind the casing was properly packed. When the drilling was in sound rock, the hole was not cased and no unusual precautions were necessary.

Frequently the drill was removed as soon as the bit had cut through the concrete slab, and a light bent, on which an air hoist was mounted, was then

placed over the hole. The over-burden was excavated by hand and hoisted to the surface in buckets, casing being set as required. When ledge rock was reached, or when large boulders were encountered, excavation was stopped and it was necessary to wait until a drill could be replaced upon the hole. The progress made by this method, especially when numerous boulders were encountered, was not equal to that obtained with the core drills but, without increasing the unit cost, it permitted the work to proceed simultaneously on a larger number of holes than would have been possible if only the drills had been used. By constantly shifting the drills to the places where they could operate to best advantage, the progress was aided materially.

Handling Heavy Flows of Water

When heavy flows of water were encountered, despite the precautionary measures taken to prevent them, it was necessary to stop drilling and to move the drill immediately to another location to avoid loss of time. The volume of flow, as judged by the pumping capacity required to handle it, varied from several hundred to several thousand gallons per minute, and prolonged pumping usually resulted in appreciably increased flow. Of the various methods that were tried for stopping these flows so that drilling might be resumed, only one proved entirely satisfactory. First, after striking a heavy flow, a 4-in. centrifugal pump was placed in the hole to learn the depth to which the surface of the water could be lowered by pumping. The size of the hole at this depth was then ascertained and the pump was removed. A laminated, circular, wooden plug, about 12 in. thick, and tapered from botton to top, was made of 3-in. boards, with tar-paper between the layers to reduce leakage. The plug was tapered in order to form a groove for calking against the steel casing, and the diameter was such as to permit the plug to just pass into the hole at the depth at which it was to be set. A 4-in. pipe passing through the center of the plug was connected to the suction of a motor-driven centrifugal pump to form the assembly shown in Fig. 7. This assembly was suspended from a small bent and lowered into the hole with the pump running. A valve in the discharge of the pump permitted regulation of the flow to avoid loss of prime, and this valve was gradually opened as the volume of the incoming water increased with the lowering of the level in the hole. When the plug had reached the intended position, the V-shaped groove formed between the plug and the casing was calked tightly with oakum backed up with wooden wedges. During the calking, and until the plug could be firmly braced against uplift, it was necessary to keep the pump in operation. The total static pressure acting to lift the plug was frequently in excess of 15 tons, and it was necessary to brace to the casing and to weld together, at the laps, the individual pieces of casing above the plug to help carry the load.

When the calking and bracing were completed, the valve in the suction was closed and the pump was removed. The pipe passing through the center of the plug was then extended to the surface and the valve was opened so that the cast-iron foot valve could be broken. A few blows with a heavy rod of reinforcing steel inserted through the suction pipe usually shattered it so that

it would offer no obstruction to the passage of grout. A pump was then connected to the pipe so that grout could be forced below the wooden plug and back into the passages through which the water had entered. A small, air-driven, centrifugal pump was always placed on top of the plug while grouting to remove any water or grout that might leak from below and to permit any additional calking that became necessary.

For grouting below the plug, an extra thick mixture with a water-cement ratio varying from 0.40 to 0.33, and containing 3% of calcium chloride by weight of the cement, was used. Once started, the grout was pumped continuously and slowly until no more could be injected at a pressure that exceeded the static head by approximately 20 lb per sq in. The pumping rate and pressure were held down in an effort to avoid wasting the grout by forcing it to distant areas where it was not needed. The calcium chloride, by accelerating the set of the cement, reduced the quantity of grout required to plug the water passages and permitted a resumption of drilling almost immediately upon the completion of the grouting. It was necessary to remove the plug by hand, and the grout filling the hole below it was either mucked out by hand, using paving breakers, or was cored out with a drill. Better progress resulted from the use of the core drills. In every case where the methods that have been described were used, the water was stopped successfully so that no further trouble was experienced in the hole, unless other water-bearing passages were intercepted at a lower elevation. In a number of cases, the water (as evidenced by the quantities of leaves and trash brought in) was entering through passages that extended to the river-bed outside of the coffer-dam. With the normal seasonal flow, the river water stood about 25 ft above the average elevation of the tops of the holes, which meant that it was necessary to work against an appreciable head when water was struck in the lower part of a hole.

It has probably become apparent that the greatest difficulties that arose in sinking the holes were encountered in the over-burden and in unsound, water-bearing rock. Where the rock was sound, and excess water was not encountered, the work became more or less routine. Sound rock was cut smoothly, with a minimum of trouble from blocking and binding, and the core was usually removed in a single piece without difficulty.

When the shale and bentonite bed previously described was passed, or sound rock was apparently reached, a jackhammer hole was drilled 8 or 10 ft into the bottom of the hole to learn whether any openings existed in this zone. A rod of 0.5-in. tool steel, with a 90° hook on the lower end, was used to feel for seams in the small hole. If the rock below was sound or contained only small seams, the large hole was extended no deeper and a pipe was calked into the jackhammer hole to permit the grouting of any small seams later. If the dip or folding of the rock caused the bottom of a hole to slope steeply, it was leveled roughly with a jackhammer.

Column Construction and Tolerances

Upon the completion of drilling, a hole was always filled with concrete as quickly as possible to avoid the constant threat of increasing leakage.

The reinforcement consisted of sixteen parallel steel bars 1 in. sq, pre-fabricated into 32-in. cages not more than 40 ft long. These cages were placed in the holes with a crane and then carefully braced in a position as nearly plumb as possible. A variation not exceeding 1.5 in. from the vertical was allowed when necessary. It was practically impossible to keep the large holes plumb while they were being drilled, as any number of conditions may cause them to turn, and it was necessary to check at frequent intervals so that any deviation might be corrected before it was too late.

A hole could sometimes be plumbed by bracing the drill rods to one side, well above the bit, so that they were thrown into a bow. This forced the bit to lean in the desired direction and the hole could occasionally be righted. More certain results were obtained by reducing the size of the bit sufficiently so that a smaller hole could be started properly. This method was found to be the best if the reduction left enough margin to insure that it would be possible to complete the hole at the required size.

After the reinforcing steel was plumbed and fastened rigidly in position by short steel bars welded to the cage and bearing against the casing, it served as a useful ladder extending the full depth of the hole. This ladder was used to advantage by the workmen while calking the numerous small leaks that usually spouted through the casing joints and for running grout pipes to any openings known to exist behind the casing. These pipes were either connected to nipples welded over holes burned through the casing, or were flattened and forced into any space left where the sections of casing lapped. Despite the efforts made to fill the space behind each section of casing, the material was occasionally washed out by the inflowing water. The grout pipes were placed in order that these spaces might be filled after the columns were poured, affording all possible lateral support for the full length of the column.

As wide intervals of time usually elapsed between the pouring of the first and the last of the columns supporting any one pier, it was feared that the varying degrees of shrinkage, which would have resulted when the columns were loaded, would result in an unequal distribution of stresses in the pier, and cooling pipes were installed to dissipate the heat generated by the setting concrete. The cooling system consisted simply of two vertical pipes connected at the bottom to form a U, through which cool water was forced after the concrete had attained its final set. Resistance thermometers embedded in the columns indicated that a decided drop in temperature resulted when water was circulated.

To fill the holes, concrete was dumped from 3-yd buckets at the surface into a small, steel hopper, with a short guide chute, mounted directly over the hole. Vibrators were used to obtain a proper density of the concrete and to insure that the steel would be well covered for the full length of the column.

Although the columns were 49 in. in diameter at the top where they passed through the slab, the diameter of the upper 18 in. was reduced to 36 in. with a pipe form. Before the first lift of the pier was poured, a cushion of bags was placed upon the offset formed at the reduction. This was done to avoid

loading the columns eccentrically, as it was rarely possible to plumb the steel reinforcement so that it was concentric with the upper part of the hole. It was not intended that the slab that served as a working platform for the drills should carry any of the load imposed by the structure.

Special Treatment of Nose Pier

The methods that have been described are those which, in general, were used for sinking the columns that support the piers carrying the guard wall. In the case of the nose pier (Pier 27, Fig. 3) it was necessary to vary the procedure that was followed in pouring the working slab for the drills. The excavation of the over-burden at this pier was begun before it was known that it would be necessary to support all of the piers upon columns, and it was soon learned that great irregularity of the rock surface characterized the area. The surface of the rock was found only a few feet below the surface over part of the area of the pier; later it was found to lie as much as 50 ft below the surface only a short distance away. Removal of the first few feet of over-burden released a number of heavy flows of water in the area of the pier, and it was impossible to determine whether it issued from passages in the rock or was traveling in a bed of gravel overlying the rock. Although steel sheet-piling was driven around the pier, the flow continued undiminished, and the great irregularity of the depths at which the piling stopped indicated that the water was probably issuing from crevices and joints through the ledge rock. The total flow into the sheet-pile enclosure was approximately 12 000 gal per min, and it was obviously not feasible to attempt to rest the pier directly upon rock.

In order to obtain a concrete slab upon which the drills could be operated, the area of the nose pier was cleaned up with clam shell and hand labor, and boxes were built around the suctions of three pumps that were left in the enclosure to handle the water. One other pump removed water from a sump excavated just outside of the enclosure. From one particularly bad boil, a wooden-box culvert was carried across the bottom to one of the pumps. Cobbles were then dumped into the water, covering the bottom of the enclosure and surrounding the boxed-in pump suctions until they rose above the water to form a level surface. The interstices between the cobbles formed passages through which the water traveled to the various pump suctions. Vertical grout pipes were installed, leading into the culvert before it was covered, and at random in the cobbles as they were placed. The surface of the cobbles was covered with small stone and the first lift of concrete was poured, using the steel pile enclosure for a form. In order to improve working conditions by bringing the level of the concrete up to that of the surrounding area, another lift was poured later, through which holes were formed for the columns.

Before the drilling for the supporting columns was begun, the pumps were removed and grout was forced into the cobble bed through the pipes that had been provided for that purpose, the wooden culvert being filled at the same time. Subsequent inspection, made when the large holes were drilled, revealed that the cobbles had been well stabilized by the grout. Each column location in

the nose pier was grouted in advance in the same manner as has been described for the other piers.

Conclusion

In selecting the methods that were chosen for obtaining a foundation for the wall, full cognizance was taken of the fact that much was unknown concerning the practicability of drilling large holes in the type of material that was to be encountered and some doubts were felt as to the economy of the choice. However, cost and progress records which have been compiled since the work was completed indicate that, of the methods considered feasible, none would have proved as economical or as expeditious as the one selected.

Using these methods, 39 holes of an average depth of 55.1 ft and a maximum depth of 82 ft were completed. A total of 2 147.2 ft was drilled of which 1 039.9 ft was in over-burden and 1 107.3 ft was in rock. During a typical operating period of one month, when the obstacles and difficulties encountered were about average, the progress amounted to 10 ft per drill per 24-hr day, or 0.417 ft per hr. At this rate of drilling, the direct labor charge for a crew consisting of driller with two helpers, mechanic with two helpers covering all drills, and foreman is $8 per lin ft. Hourly rates of pay were as follows: Foreman $1.25, drillers $1, mechanics $1.10, and helpers 75 cents and 60 cents. Power, water, air, shot, casing, shop repairs, other services, depreciation of drilling equipment, and overhead should exceed these values somewhat, and a total cost of between $18 and $20 per lin ft is approximately correct. Under different conditions, where the drilling could be done from the surface of the rock, and where the hazard of excessive flows of water did not exist to make casing necessary, it should be possible to reduce the unit cost appreciably.

Observations made since the piers were fully loaded have not disclosed any evidence of settlement of the foundations or of deflection of the guard wall that is carried by the piers.

The operations described herein were conducted 24 hr per day in four 6-hr labor shifts and, despite the numerous delays that were suffered when water broke into the holes and made a cessation of drilling necessary, the work was completed in time to permit the lock to be opened for navigation when scheduled.

Acknowledgments

The Chickamauga Project Navigation lock was built by the Tennessee Valley Authority with its own forces: T. B. Parker, M. Am. Soc. C. E., Chief Engineer of the Tennessee Valley Authority, Lee G. Warren, M. Am. Soc. C. E., Project Engineer, with James B. Hays, M. Am. Soc. C. E., Construction Engineer, and Mr. F. C. Schlemmer, Construction Superintendent. Mr. H. R. Johnston was in charge of the work described at its inception and was succeeded by the writer. Jack C. Evans, Jun. Am. Soc. C. E., Mr. G. A. Carlson, and Mr. H. E. Murray each supervised the operations on 8-hr shifts.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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REPORTS

WIND BRACING IN STEEL BUILDINGS

SIXTH PROGRESS REPORT OF SUB-COMMITTEE NO. 31 COMMITTEE ON STEEL OF THE STRUCTURAL DIVISION

Introduction

Since the publication of its Fifth Progress Report[1] the Sub-Committee has directed its attention chiefly to a consideration of the following: (*A*) Approximate methods of determining the column wind reactions from a consideration of member stiffnesses only; (*B*) the effect of direct deformation in the columns on the stresses in a wind bent; (*C*) the torsional effects of wind on buildings; and (*D*) the magnitude of the assumed wind force on tall buildings.

(*A*) Determination of Column Wind Reactions from Member Stiffnesses

Development of Methods of Wind Stress Analysis.—Before the introduction of the slope-deflection method of analysis, the accepted procedure for the determination of wind stresses in building frames of the usual type, with small but rigid bending connections, was first to assume arbitrarily that the frame would act in some predetermined manner, thus:

(*a*) As a portal, with no vertical wind reactions or direct wind stresses at interior columns;

(*b*) As a cantilever, with column wind reactions and total wind stresses proportioned to the distance of the column from the neutral axis of the bent; and,

(*c*) In some manner as a compromise between Items (*a*) and (*b*).

It was next assumed that points of contraflexure of all members were at mid-length, and all wind moments were then computed by purely statical methods, making $\Sigma M = 0$ at each joint.

The slope-deflection method, introduced in 1915 by W. M. Wilson and G. A. Maney,[2] Members, Am. Soc. C. E., provided a theoretical type of analysis which was recognized as accurate, with certain limitations; but its

Note.—This report was presented at the meeting of the Structural Division, New York, N. Y., January 19, 1939. Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **October 15, 1939.**

[1] *Proceedings*, Am. Soc. C. E., March, 1936, p. 397.

[2] *Bulletin No. 80*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill.

application was so laborious that it was not practicable to apply the method very extensively to tall buildings.

In 1929 C. T. Morris, M. Am. Soc. C. E., and Mr. A. W. Ross, Jr.[3] presented an abbreviated method of applying the slope-deflection principles, which became known as the "Ross Method." It led to acceptably approximate results, but still involved much time and labor.

In 1930 Hardy Cross,[4] M. Am. Soc. C. E., published his method of end-moment distribution which was accurate to the same degree as the slope-deflection method and was incomparably shorter and less laborious than the latter. However, it still involved a long and tedious process, particularly for tall towers. The method has since been abbreviated in various ways, notably in 1933 by L. E. Grinter,[5] M. Am. Soc. C. E.

In 1930 Henry V. Spurr,[6] M. Am. Soc. C. E., introduced what is known as the "Spurr Method," the outstanding features of which were:

(1) The elimination of the necessity for a complete preliminary design;

(2) The justification of ignoring the secondary moments due to the change in length of columns under direct wind stress; and

(3) The control of the deflection of the bent within desired limits.

Condition (2) required that the bent act as a vertical cantilever under wind force, and the design was made in accordance with this assumption, eliminating secondary moments.

Influence of Girder Stiffnesses on Wind Reactions.—In January, 1936, F. P. Witmer, M. Am. Soc. C. E., in collaboration with Mr. Harry H. Bonner,[7] published results of a study with mechanical models which showed the trend of vertical wind-reaction ratios for symmetrical four-column bents under varying relations as to dimensions of bent, sizes of girders, and sizes of columns. It was shown conclusively that it was unjustifiable to assume a predetermined relation between the vertical wind reactions at columns without also insuring that the relative size of girders in the different bays, and of inner and outer columns, was such as would produce this assumed relation of reactions. The wind-reaction ratio—that is, the ratio of vertical wind reaction of inner column to that of outer column—was seen to be wholly determined by the relative sizes of members and to be entirely under control by properly varying these relative sizes.

In November, 1936, Professor Witmer[8] showed that, if a bent of any number or width of bays and any number or height of stories was designed for vertical floor loads alone, with certain observed conditions, the vertical reactions and direct stresses for interior columns due to any assumed wind load were always mathematically equal to zero, thus conforming exactly with the original portal theory of bent behavior. For the realization of cantilever

[3] *Bulletin No. 48*, Ohio State Univ. Eng. Experiment Station, Columbus, Ohio.

[4] *Proceedings*, Am. Soc. C. E., May, 1930, p. 919 (*Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1).

[5] Presented at meeting of Structural Division, Am. Soc. C. E., January 19, 1933 (*Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 610).

[6] "Wind Bracing," by Henry V. Spurr, McGraw-Hill Book Company, Inc., New York, N. Y.

[7] *Proceedings*, Am. Soc. C. E., January, 1936, p. 3 (*Transactions*, Vol. 102 (1937), p. 416).

[8] *Loc. cit.*, November, 1936, p. 1496.

action, special relations of sizes must be provided; and, for a general case with sizes assigned without consideration of their relative magnitude, the vertical wind reactions are absolutely unknown and, so far as is known, are unpredictable by any method thus far proposed, short of a rigid theoretical analysis.

Value of Determining Wind Reactions First.—A fundamental desideratum in the analysis of any framed structure is the preliminary determination of a complete set of balanced external forces and reactions. This order of procedure is habitually reversed in the analysis of a building bent by slope-deflection or moment distribution, the vertical wind reactions not being found until after all girder moments have been computed.

In such a bent, if these vertical reactions can first be found correctly, the stresses may be analyzed quickly, with reasonable accuracy, by statically determinate processes. In fact, as stated before, this method of procedure has long been followed where the bent has been assumed to act in accordance with some theory such as the portal or the cantilever theory. The weakness of this procedure lay in the fact that the behavior was arbitrarily assumed without regard to the sizes of the members, and therefore with only a guess as to the value of the reactions that would result from the application of wind forces.

Notation.—The letter symbols introduced in this report conform essentially to American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials[9] compiled by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932.

Reaction-Ratio Curves for Symmetrical Four-Column Bents—With this thought in mind, Professor Witmer followed up his model study by the theo-

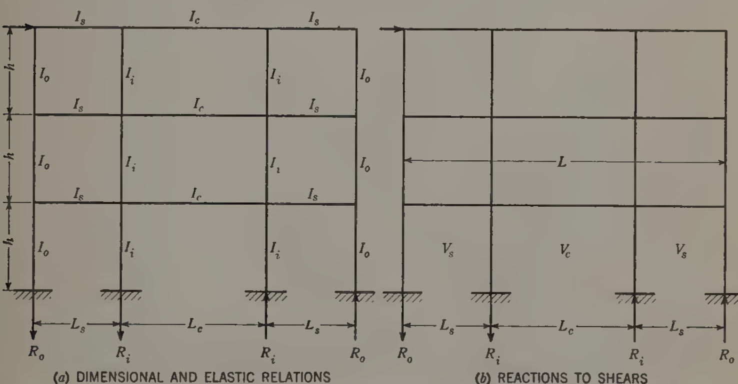


FIG. 1.—RELATION BETWEEN VARIOUS CHARACTERISTICS FOR SYMMETRICAL, FOUR-COLUMN BENTS

retical consideration of a large number of bents representing a great variety of proportions. Using the Cross method of analysis he computed the wind-

[9] ASA—Z10a—1932.

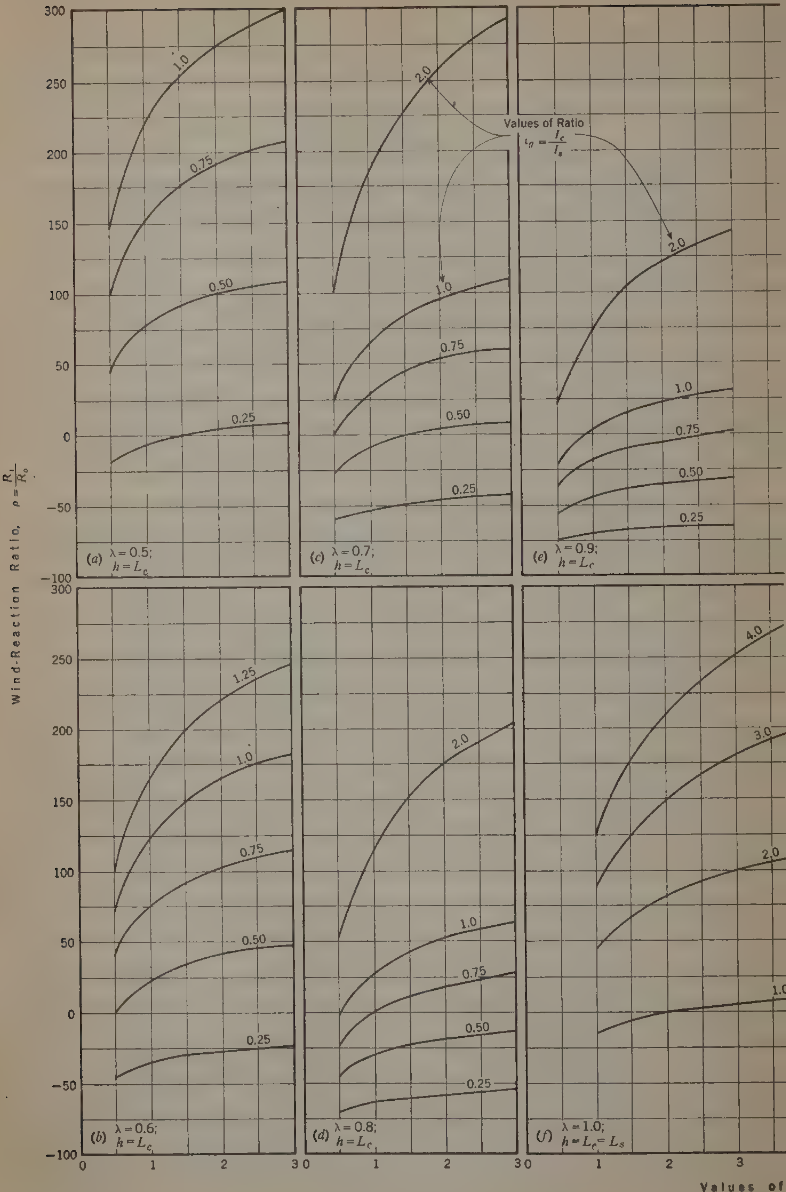
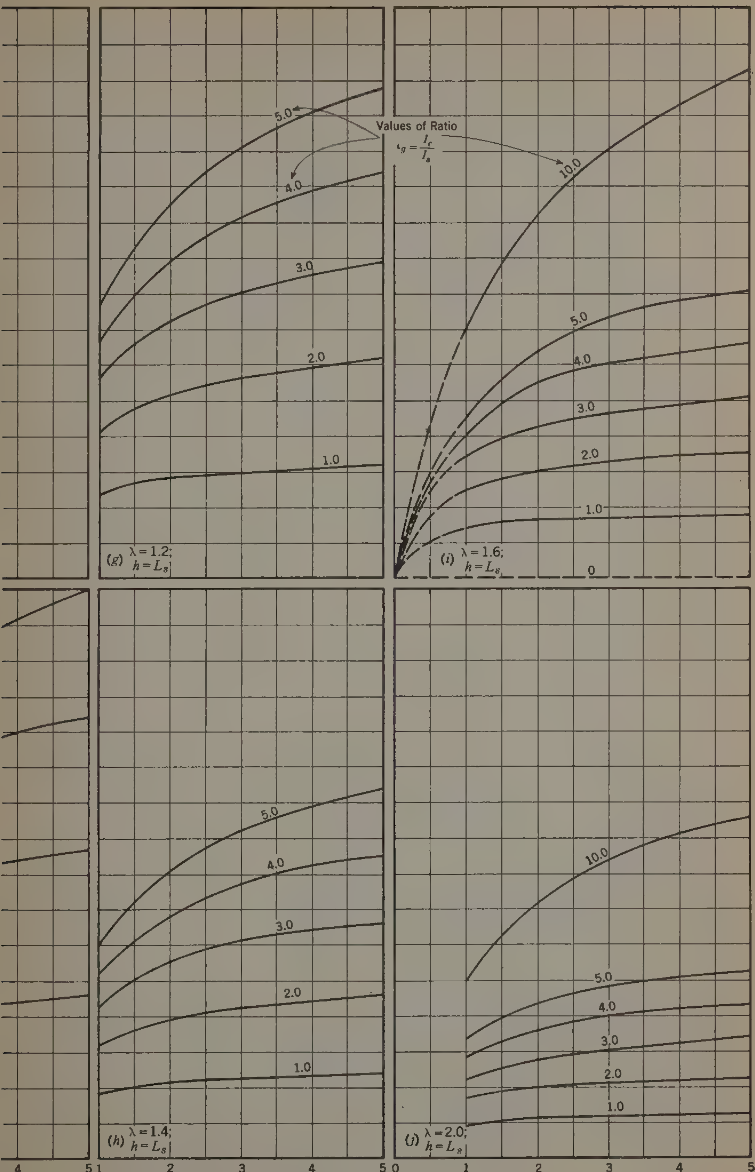


FIG. 2.—VALUES OF



Ratio, $\iota_c = \frac{I_i}{I_o}$

WIND-REACTION $\rho = \frac{R_i}{R_o}$

reaction ratio in each case. A series of curves were thus evolved by which, knowing the actual sizes of all members of a symmetrical four-column bent, the reaction ratio may be determined quickly with great accuracy.

The diagrams are constructed with reference to certain dimensional and elastic relationships, which (referring to Fig. 1(a)) are:

$$\lambda = \frac{L_c}{L_s} = \frac{\text{width of center bay}}{\text{width of side bay}} \qquad (1a)$$

$$\iota_g = \frac{I_c}{I_s} = \frac{\text{average moment of inertia of all center girders}}{\text{average moment of inertia of all side girders}} \qquad (1b)$$

$$\iota_c = \frac{I_i}{I_o} = \frac{\text{average moment of inertia of all inner columns}}{\text{average moment of inertia of all outer columns}} \qquad (1c)$$

and,

$$\rho = \frac{R_i}{R_o} = \frac{\text{vertical reaction inner column}}{\text{vertical reaction outer column}} = \text{wind reaction ratio} \qquad (1d)$$

Fig. 2 gives the values of the reaction ratio for values of λ varying from 0.5 to 2.0 and for various values of ι_g and ι_c. In constructing the curves, it was assumed that the average story height h was equal to the smaller bay width, L_s or L_c. Referring to Fig. 1(b), let ρ' and ρ'' be two differing values of ρ. Let V_s' and V_c' be vertical wind bay shears for ρ' and let V_s'' and V_c'' be vertical wind bay shears for ρ''. Then, if L = total width ($= 2\,L_s + L_c$), it can be shown easily that

$$\frac{V_s'}{V_s''} = \frac{L + \rho''\,L_c}{L + \rho'\,L_c} \qquad (2a)$$

and,

$$\frac{V_c'}{V_c''} = \frac{V_s'}{V_s''}\left(\frac{1+\rho'}{1+\rho''}\right) \qquad (2b)$$

By applying Equations (2) it was then shown that, for $h = 1\frac{1}{2}$ times, or $h = \frac{2}{3}$ of, the value assumed, the results by the curves were within permissible limits of accuracy.

It was also assumed that the average moment of inertia of the outer column, I_o, was the same as that of the outer girder, I_s. These relations may vary considerably, however, without greatly affecting the value of the reaction ratio; so that practically any reasonable relations of girders and columns will yield satisfactory results with the curves.

It is interesting to note that, if $\iota_c = 0$, $\rho = -100\%$, for all values of λ and ι_g; and if $\iota_g = 0$, $\rho = -100\%$ for all values of λ and ι_c. These conditions may be shown readily to be necessary mathematically (see Fig. 2(i)).

Extreme accuracy in the value of ρ is not necessary. Assuming $\rho' = +85\%$; $\rho'' = +98\%$; and $L = 3.3$: Then, $L_c = 1.3$; $L_s = 1.0$; and, $\lambda = \frac{1.3}{1.0} = 1.3$. Substituting these values in Equations (2), $\frac{V_s'}{V_s''} = 103.8\%$, and $\frac{V_c'}{V_c''} = 97.0\%$. Hence, for the case shown, although ρ' and ρ'' differ

from each other by about 15%, the corresponding values of vertical shears for the center bay differ by only 3.8%, and for the side bay by only 3.0%; and, the resulting girder moments will thus differ only by these last-named percentages.

Basis of Witmer Method of K-Percentages.—The curves of Fig. 2, however, are not applicable to bents of unsymmetrical form or of more or less than four columns. Further study by Professor Witmer led to a method for determining these ratios which may be applied generally to any bent for which the sizes are known throughout. The reaction ratios for the various columns are found to depend wholly upon the stiffness factors $\left(K = \frac{I}{L}\right)$ of the members and can be found readily by a consideration of these quantities.

In the analysis of a bent by the Cross method, it is found that the ratios of vertical wind shears in the different bays change in value very slightly from cycle to cycle. They are practically the same after the first cycle as after a complete Cross analysis. Going back one step further, one may derive vertical shear ratios which are sufficiently close for stress computation by the stiffness factors of girders and columns without even performing a single cycle of the Cross analysis.

It was also felt that the ratios of total vertical wind shears in the different bays for any wind load should bear a definite relation to the ratios of the sums of moments of inertia of all girders in the respective bays, since in any bay the total vertical wind shear is taken by all the girders in that bay. This hypothesis was verified by investigating a large number of cases with widely varying proportions. Furthermore, it was found that the vertical shear ratios for any bent were practically the same as for a simple, compromise, three-story bent in which girders and columns were assumed to have sizes equal to the average of those of the respective members in the actual bent. Thus, any given bent may be quickly reduced to a three-story compromise bent, and the wind-reaction ratios determined therefrom.

Here again, a simplification may be introduced by what may be termed a method of "K-percentages." This method is as follows:

(1) Find the average moment of inertia, I, for all girders in each bay, separately, and also the average value of I for the full height of each column of the bent;

(2) Assume a three-story compromise bent, using the average sizes found in Step (1) for the respective members and, for all stories, using a height equal to the average story height of the bent;

(3) At each end of each girder in this compromise bent, place the percentage which its stiffness factor K bears to the sum of the K values of all members meeting at the joint (percentages for columns need not be considered);

(4) Increase the K-percentages of girders at any column in the proportion that the average K-value of that column bears to the average K-value of the left-hand, outside column (these corrected percentages may be regarded as representing bending moments in girders due to a hypothetical wind force applied at the top floor of the compromise bent); and,

(5) Add the corrected K-percentages separately in each bay and divide by the width of the bay. The quotients may be regarded as the total vertical shears in the respective bays from the hypothetical wind force before mentioned. From these shears the corresponding vertical reactions at all columns and, from these, the wind-reaction ratios for the bent, may be found readily.

Stress Analysis When Reactions Are Known.—Having found the wind-reaction ratios for all columns, the stresses may be found by a statically determinate process as follows:

(*a*) Assume all points of contraflexure at mid-length of member, except for columns in the first story, where they may better be assumed at six-tenths of the story height above the base;

(*b*) Find the vertical wind reactions and the direct wind stresses in columns by writing a series of moment equations of the assumed wind forces about the points of contraflexure of the right-hand, outside column, in the various stories of the bent (in these equations the column wind reactions or direct wind stresses will be expressed in terms of those of the left-hand, outside column by means of the wind-reaction ratios);

(*c*) Find the girder shears by considering the difference between the direct stresses in the columns immediately above and below a girder at each joint as a shear increment;

(*d*) Multiply the total shear in each girder by one-half the length of the girder to obtain the girder moment at each end;

(*e*) Determine the column moments that will make $\Sigma M = 0$ at each joint, assuming the same moment at top and bottom of any story column except for the first story (the determination of column moments will progress story by story from the top of the bent to the top of the first story); and,

(*f*) Assume that the total horizontal shear in the first story is divided among the columns in proportion to their average K-values. The total moment in any column of the first story will then equal its shear multiplied by the story height, and its base moment will equal this total moment, minus the moment already found at the top of the story. The base moments thus found will generally modify, somewhat, the location of the points of contraflexure as originally assumed for the first story; but the effect upon the vertical reactions and direct column stresses will be negligible.

If desired, instead of Step (*f*), a Cross analysis may be applied readily to the first story alone.

Examples of Application of the Method.—Analysis by the foregoing method is very quickly performed. Results obtained by it were variously compared with those by slope-deflection,[2] the Grinter method,[5] the Goldberg method,[10] and the Cross method[4] in the following cases, representing a great variety of proportions and reaction ratios:

(1) Wilson-Maney twenty-story, four-column bent (reaction-ratio $\rho = +32.3\%$);

[10] "Wind Stresses by Slope Deflection and Converging Approximations," by John E. Goldberg, Jun. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 962.

(2) Ten-story, four-column bent, University of Pennsylvania design ($\rho = +10.5\%$);

(3) Twenty-story, four-column, unsymmetrical bent designed by Spurr method[6] (University of Toronto design);

(4) Nine-story, five-column, unsymmetrical bent, University of Pennsylvania design;

(5) American Insurance Union Building, Columbus, Ohio; thirty-eight-story, six-column tower, computed below the twenty-sixth floor ($\rho = +44.8\%$ for second column and -60.6% for inner column); and,

(6) Chateau Crillon, Apartment Building, Philadelphia, Pa.; twenty-seven story, three-column bent ($\rho = +4.6\%$).

Comparisons of the results of applying the method to these cases with the results obtained by other methods are given herein. Because the proposed method necessitates an assumption of a central location for the point of contraflexure (except as specified in the bottom story), it is thought reasonable to compare moments found thereby with the average of the moments in the member as found by theoretical analysis. In other words, the method is considered justified if the total moment distributed by it to any member is nearly enough in accord with the total moment distributed to that member by rigid theory.

Avoidance of Secondary Moments.—The method described does not include secondary moments due to change in length of columns from direct wind stress. An interesting further development, however, is the ease with which there may be determined, in advance of design, the proper relative summations of moments of inertia of girders in the respective bays in order that an assumed wind force may actually develop vertical reactions in accord with the cantilever theory, thus making it permissible to ignore these secondary moments. The procedure in this case is as follows:

Having found the necessary relative vertical wind reactions by properly considering the areas of the columns required for vertical loads, in conjunction with the locations of the columns in the bent, the relative, total, vertical wind shears in the different bays readily follow.

If, now, the vertical wind shear in each bay is multiplied by the width of the bay, the results in the different bays will be approximately proportional to the total sums of required K-values for girders in the respective bays; and if these quantities are again multiplied by the respective widths of the bays, the products will be approximately proportional to the total sums of moments of inertia required in the girders of the respective bays. More briefly stated:

$$\Sigma I \text{ varies as } V_g\, L_g^2$$

in which ΣI = approximate sum of moments of inertia of all girders in a bay; V_g = relative vertical wind shear in a bay as determined from relative wind reactions required by the cantilever theory; and, L_g = length of girder in the bay.

Application of Spurr Method Facilitated.—With these relations of I in the several bays known, design by the principles of the Spurr method may be

greatly facilitated. If the computed deflection of the building as thus designed is found to be excessive, it may be corrected by a proportional decrease in column unit stresses, which is the same as a proportional increase in column areas, since the cantilever reaction relation is not disturbed. However, the method is generally uneconomical. Since the larger part of the total deflection is due to girder bending, it is generally better to select stiffer girders to minimize deflection. An application of this procedure is given subsequently in Fig. 5.

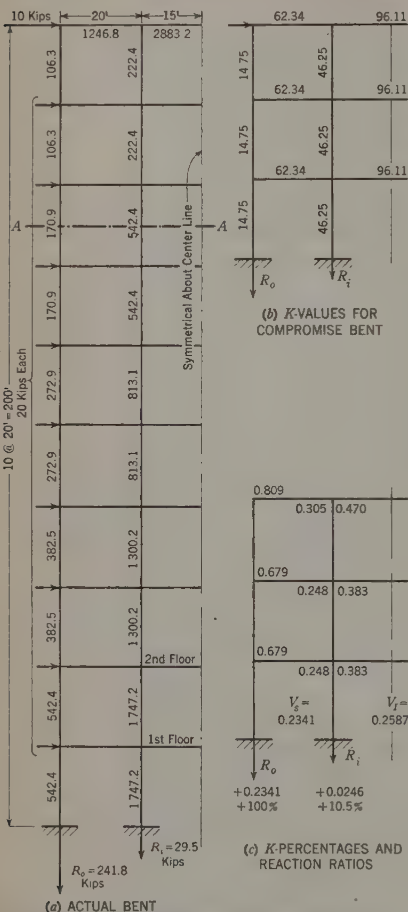


Fig. 3.—Case (2); Analysis of Symmetrical Bent Designed at University of Pennsylvania

Accuracy Attainable by Method of K-Percentages.—It is important to note that a considerable variation may occur in the value of the reaction ratio used for computation by the foregoing method without materially changing the relative values of the bay shears. Thus, a variation of 10% or more in the value of ρ will generally produce not more than 2% to 3% change in the bay shear and the consequent girder moments. This has already been demonstrated mathematically.

To summarize, in conclusion, the proposed method is similar to that which was formerly in use, before the rigid theoretical types of analyses were presented, but with one vital difference: Whereas formerly the stresses were based upon a guess as to the manner in which the bent would act under wind, with consequent uncertainty as to the value of the vertical wind reactions from which the stresses were computed, the proposed method is founded upon reactions rationally determined so that they will be closely in accord with the actual sizes of the members in the bent. The resultant stresses, therefore (while frankly approximate, owing to assumptions as to the location of the points of contraflexure), are reasonably in agreement with these reactions, and

therefore may be relied upon to be generally not too greatly in error. If greater accuracy is insisted upon, it can only be assured by the tedious and time-consuming methods of rigid theoretical analysis; and, it is questionable whether this is generally justified in view (1) of the uncertainty as to the actual intensity and distribution of the wind forces and (2) of the fact that wind stresses are customarily neglected unless they are greater than some specified proportion (usually about 25%) of the stresses from vertical loads.

An idea of the degree of accuracy attainable by the method of K-percentages may be gained through the consideration of the results obtained by applying the method to the six cases previously mentioned.

Three-Bay, Ten-Story, Symmetrical Bent.—A characteristic simple example of the application of the method is the case of a three-bay, ten-story, symmetrical bent designed at the University of Pennsylvania (Case (2)) as indicated in Fig. 3. The moments of inertia for the columns are given in Fig. 3(a), the values of I_s and I_c being 1 246.8 and 2 883.2, respectively, throughout. For this case ι_g (Equation (1*b*)) = 2.31, and ι_c (Equation (1*c*)) = 3.14. By the Cross analysis, ρ (Equation (1*d*)) = + 12.2 per cent.

Computations for V and R are arranged as follows:

$$\begin{array}{llll}
0.305 & & 0.809 & \\
0.248 & & 0.679 & \\
\underline{0.248} & & 0.679 & \\
0.801 & \times 3.14 = & \underline{2.515} & \\
 & & \dfrac{4.682}{20} & = 0.2341 = V_s = R_o
\end{array}$$

$$\begin{array}{ll}
0.470 & \\
0.383 & \\
\underline{0.383} & \\
1.236 & \times 2 \times 3.14 = \dfrac{7.762}{30} = 0.2587 = V_c
\end{array}$$

Finally, $R_i = V_c - V_s = 0.0246$; and, the reaction ratio ρ (Equation 1(*d*)) = + 10.5 per cent. The Cross analysis of a five-story compromise bent yields ρ = + 8.2 per cent.

Assuming the plane of contraflexure, Section A-A, as at mid-story height of the columns for all but the bottom story, and at six-tenths of the story height above the base for the bottom story, and equating the overturning moment about this plane to the moment of resistance of the column stresses, it follows that $R_o = \dfrac{M_A}{73.15}$.

Since the reaction ratio for an interior column is 0.105, the vertical shear in the outer bay is equal to R_o and in the inner bay it is 1.105 R_o. If the points of contraflexure of all girders are assumed at mid-span, the maximum moment in the outer girders of the top floor is

$$M_o = \frac{M_{10}}{73.15} \times 10 \ldots\ldots\ldots\ldots\ldots\ldots (3a)$$

and in the inner girder

$$M_i = \frac{M_{10}}{73.15} \times 1.105 \times 15 \ldots\ldots\ldots\ldots\ldots\ldots (3b)$$

For a lower floor, such as the ninth, the maximum girder moment is, for an outer girder:

$$M_o = \frac{M_9 - M_{10}}{73.15} \times 10 \ldots\ldots\ldots\ldots\ldots\ldots (4a)$$

and for an inner girder,

$$M_i = \frac{M_9 - M_{10}}{73.15} \times 1.105 \times 15 \ldots\ldots\ldots\ldots\ldots\ldots (4b)$$

The actual overturning wind moments do not need to be computed. If H_9 and H_{10} are the horizontal shears in the ninth and tenth stories respectively, and h_9 and h_{10} are the story heights, then

$$(M_9 - M_{10}) = 0.5\, H_9\, h_9 + 0.5\, H_{10}\, h_{10} \ldots\ldots\ldots\ldots (5a)$$

For the first floor,

$$(M_1 - M_2) = 0.4\, H_1\, h_1 + 0.5\, H_2\, h_2 \ldots\ldots\ldots\ldots (5b)$$

From Equations (3) and (4) the girder moments M_k given in Table 1 were

TABLE 1.—COMPARATIVE MOMENTS IN SYMMETRICAL THREE-BAY, TEN-STORY BENT (CASE (2))

Floor or Story	Girders: Cross Analysis, Left	Girders: Cross Analysis, Right	Girders: Witmer; M_k	Girders: Percentage difference	Columns: Cross Analysis, Left	Columns: Cross Analysis, Right	Columns: Witmer; M_k	Columns: Percentage difference
(a) OUTER MEMBERS								
10	16.3	14.9	13.7	−14	16.3	14.8	13.7	−14
9	63.4	57.6	54.7	−11	48.6	47.4	41.0	−17
8	110.4	110.8	109.4	− 1	62.9	61.8	68.4	+ 9
7	151.7	167.0	164.1	+ 3	90.0	87.3	95.7	+ 7
6	209.5	224.4	218.7	+ 1	122.2	118.0	123.0	+ 2
5	266.5	280.3	273.4	0	148.4	145.5	150.4	+ 2
4	314.0	339.1	328.1	0	168.4	164.7	177.7	+ 6
3	359.3	390.3	382.8	+ 2	194.5	188.2	205.1	+ 7
2	417.9	445.6	437.5	+ 1	229.7	227.0	232.4	+ 2
1	435.6	462.1	440.3	− 2	208.6	267.9	207.9* 251.5†	− 4
(b) INNER MEMBERS								
10	20.8		22.7	+ 9	33.6	33.3	36.4	+ 8
9	79.5		90.6	+14	103.7	100.2	109.0	+ 6
8	182.3		181.3	− 1	192.8	182.5	181.7	− 3
7	281.0		271.9	− 3	265.5	257.2	254.3	− 3
6	368.7		362.5	− 2	333.9	323.9	326.9	− 1
5	453.5		453.1	0	409.9	396.1	399.6	− 1
4	551.5		543.8	− 1	499.5	472.2	472.3	− 3
3	649.7		634.4	− 2	567.6	549.6	544.9	− 3
2	729.4		725.0	− 1	625.3	618.0	617.6	− 1
1	753.5		729.5	− 3	597.6	825.9	552.2* 888.4†	+ 1

* Left. † Right.

computed. Column moments, except at the base, were found by considering $\Sigma M = 0$ at each joint. The total moment in an outer column of the bottom story, using the average K's for the compromise bent, is $3\,800\,000 \times \dfrac{14.75}{122}$ $= 459\,400$ ft-lb, and in an inner column it is $3\,800\,000 \times \dfrac{46.25}{122} = 1\,440\,600$ ft-lb. The moment at the base of a column, therefore, is the total moment in the column less the moment found at the top of the story from the preceding procedure. The reaction ratio is + 10.5 per cent.

Examining Table 1 it is seen that the difference in girder moments found by the method of K-percentages and by the Cross method is generally much less than 10% and in no case more than 14 per cent. The maximum difference for column moments is 17 per cent. The largest differences are near the top of the frame where the wind moments are small in any case.

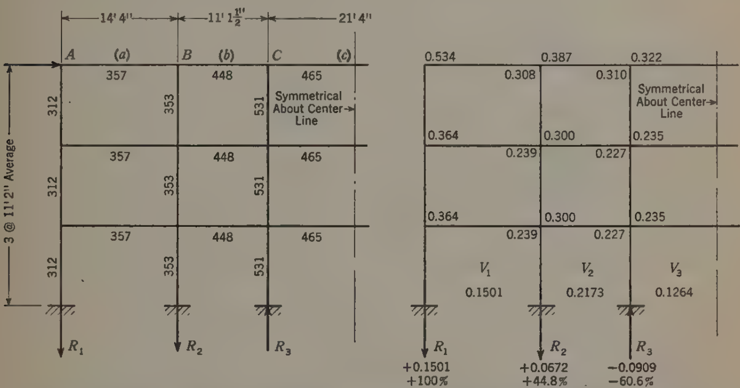


FIG. 4.—CASE (5); ANALYSIS OF SYMMETRICAL BENT; AMERICAN INSURANCE UNION BUILDING, COLUMBUS, OHIO

Five-Bay, Twenty-Six-Story, Symmetrical Bent.—The method of K-percentages was tested by application to the bottom twenty-six stories of the American Insurance Union Building in Columbus (Case (5)), as indicated in Fig. 4. The total height is thirty-nine stories, but the analysis was applied only to the part where the set-back for the outside bays occurs. This com-

TABLE 2.—CHARACTERISTICS OF SYMMETRICAL BENT; FIGURE 4

Description	Girders in Bay:			Columns:		
	(a)	(b)	(c)	A	B	C
Sum of moments of inertia..........	173 971	189 448	387 379	118 637	153 846	231 103
Number of floors....................	34	38	39	34	39	39
Average moment of inertia...........	5 116	4 986	9 932	3 490	3 943	5 926

prises a series of five-bay, six-column bents, with center girders omitted in the mezzanine and second floors. In the compromise bent, these center girders were assumed to be in place, temporarily, with $I = 25\,000$. The end moments found for these girders were then distributed by a Cross analysis and the girders omitted, the analysis comprising only a part of the fifth story. Other characteristics are shown in Table 2. Computations for V and R are as follows:

$$\begin{array}{lcl} 0.308 & & 0.534 \\ 0.239 & & 0.364 \\ \underline{0.239} & & 0.364 \\ 0.786 \times \dfrac{353}{312} & = & \underline{0.889} \\ & & \dfrac{2.151}{14.33} = 0.1501 = V_1 = R_1 \end{array}$$

$$\begin{array}{l} 0.310 \\ 0.227 \\ \underline{0.227} \\ 0.764 \times \dfrac{531}{312} = 1.3002 \end{array}$$

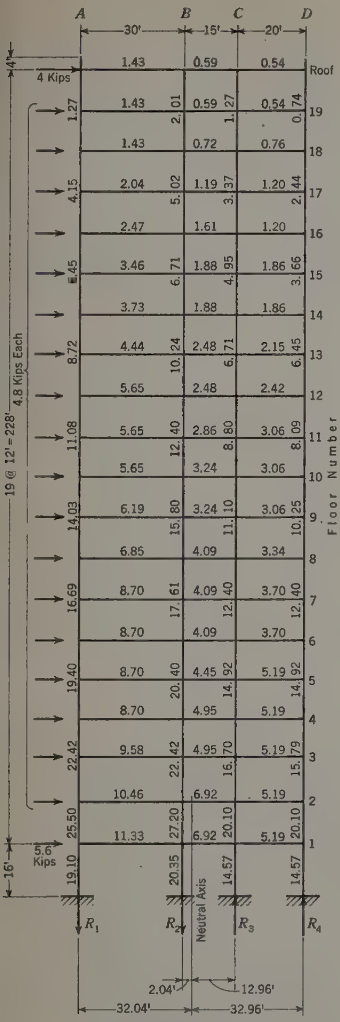
$$\begin{array}{lcl} 0.387 & & \\ 0.300 & & \\ \underline{0.300} & & \\ 0.987 \times \dfrac{353}{312} & = & \underline{1.1176} \\ & & \dfrac{2.4178}{11.12} = 0.2173 = V_2 \end{array}$$

$$\begin{array}{l} 0.322 \\ 0.235 \\ \underline{0.235} \\ 0.792 \times 2 \times \dfrac{531}{312} = \dfrac{2.6960}{21.33} = 0.1264 = V_3 \end{array}$$

For the girders in Bay (*a*), Fig. 4, the difference between the moments obtained by the method of K-percentages and the Cross method was generally less than 14% but increased to 18% at a floor about two-thirds of the height of the bent from the bottom and to 20% at the top. For the girders of Bay (*b*) the maximum difference was 20%, which was at about two-thirds of the height from the bottom; and for the girders of Bay (*c*) the maximum difference was 31% at the first-floor level, although a difference of 29% was found at about two-thirds of the height from the bottom.

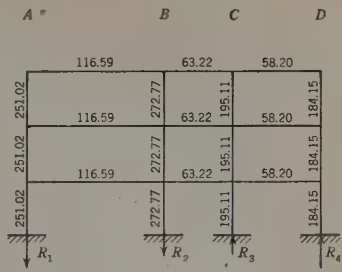
The maximum difference found for any of the columns was 23%; but in all of them differences of 14% to 22% were found at levels from two-thirds to three-quarters of the height from the base. It is believed that the rather large differences for some members may be attributed to the irregularity of the frame.

Three-Bay, Twenty-Story, Unsymmetrical Bent.—A three-bay, twenty-story, unsymmetrical bent, designed by the Spurr method at the University of Toronto (Case (3)), and shown in Fig. 5, furnished an excellent opportunity to test the applicability of the method to unsymmetrical frames. The results

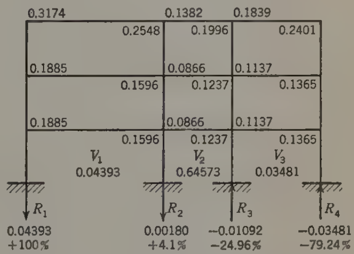


Quantities Given For Members are K-Values For Spurr Design

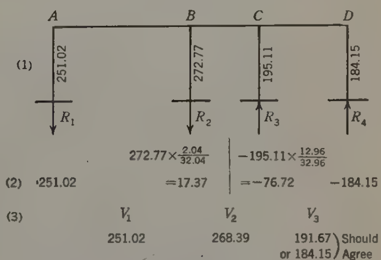
(*a*) ACTUAL BENT



(*b*) K-VALUES FOR COMPROMISE BENT



(*c*) K-PERCENTAGES AND REACTION RATIOS



Line (1) - Relative Column Areas, Assuming $A \backsim K$
Line (2) - Relative Column Reactions for Cantilever Theory
Line (3) - Relative Vertical Shears in Bays

Bay	VL^2	ΣI's From Spurr Design
A-*B*	$251.02\times30^2=225\,900$ (1)	41 970 (1)
B-*C*	$268.39\times15^2=60\,380$ (0.267)	11 376 (0.271)
C-*D*	$191.67\times20^2=76\,660$ (0.339)	13 968 (0.333)

(*d*) ESTIMATE OF REQUIRED TOTAL I's PER BAY

FIG. 5.—CASE (3); ANALYSIS OF UNSYMMETRICAL BENT DESIGNED AT UNIVERSITY OF TORONTO

were compared with those obtained by the Goldberg method[10] with generally close conformity. For girders, the divergence was less than 10% in most cases, although it was found to be 30% for one roof girder. For columns, the conformity was similarly close, although for the top story it rose, in one instance, to 138 per cent. The wind moments were insignificant here, however, and large discrepancies are of no practical consequence.

Three-Bay, Twenty-Story, Wilson and Maney Symmetrical Bent.—When applied to the classical twenty-story, Wilson and Maney bent (Case (1)), the results obtained by the method of K-percentages differed from those obtained by the slope-deflection analysis by generally less than 10% for both girders and columns. A difference of 26% was found for one roof girder and one of 18% for another. The maximum difference for a column was 13% for the outer column in the bottom story. A Cross analysis for the first story showed this difference to be 9 per cent.

Four-Bay, Nine-Story, Unsymmetrical Bent.—For a four-bay, nine-story, unsymmetrical bent (Case (4)), designed at the University of Pennsylvania, the differences between the average moments found by the method of K-percentages and by the Cross method were never more than 10% for girders and 12% for columns.

Two-Bay, Twenty-Nine-Story, Unsymmetrical Bent.—Applied to the two-bay, twenty-nine-story frame of the Chateau Crillon, Philadelphia (Case (6)), the difference between the results obtained by the method of K-percentages and by the Cross method was generally moderate. Except for the upper four stories, which were greatly complicated by a penthouse for which data were not available, the differences (excluding doubtful results) were generally less than 10%, with maximum values of 19% for girders and 36% for columns, both of these being in the two lower stories.

(*B*) Effect of Direct Deformation in Columns on Wind Stresses

General.—In a discussion of the Witmer method of K-percentages submitted to the Sub-Committee, George E. Large, Assoc. M. Am. Soc. C. E., has drawn attention to the importance, in tall buildings, of axial length changes of the columns due to wind if the floor joints do not remain in line. Relative up-and-down displacement of them accumulates from story to story, sometimes causing appreciable secondary corrections to the primary girder shears as usually computed. It is agreed that no such secondary stresses can exist if the bent has been designed by the elastic cantilever (Spurr) method, although in certain cases this method may not be the most economical. When a relatively slender wind bent is found to have primary girder shears that are inconsistent with a straight-line variation of column unit stress, the bent should be examined further to determine, if possible, the girder-shear corrections induced in the bent due to accumulated floor-joint displacement. The algebraic sum of each girder-shear correction, and the previously computed primary girder shear, will give the true final girder shear, from which the true-column, direct stresses may be computed by a summation downward from the top of the bent.

Obviously, any comparison of a design made by the portal method, or any other non-elastic method, with one made by the Spurr method is somewhat unfair unless the aforementioned stress corrections are made, together with a calculation of deflections in both cases. A method of making these girder-shear corrections has been evolved by Professor Large,[11] involving an adaptation of the Cross moment-distribution method to the distribution of column direct deformations up and down the bent.

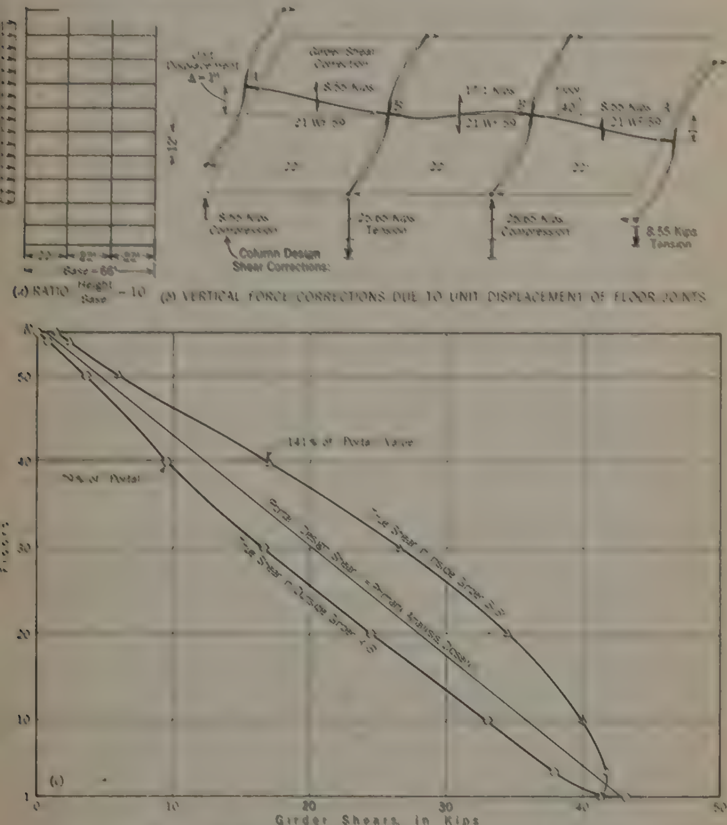


FIG. 6.—PRIMARY GIRDER SHEARS, CORRECTIONS, AND TRUE FINAL SHEAR VALUES

Results for a Fifty-Five-Story Bent.—Fig. 6(*a*) shows the analysis of a fifty-five-story symmetrical bent with three 22-ft bays and a height-base ratio of 10. It had been designed by the portal method, as is apparent in Fig. 6(*b*), for the

[11] "Settlement Stresses in Continuous Structures," by George F. Large, *Bulletin* No. 238, Ohio State Univ. Eng. Experiment Station, Columbus, Ohio.

fortieth floor, where 21-in. WF 59-lb girder sections are shown throughout. After the primary girder shears had been found, assuming contraflexure points as at mid-story height, the outside floor joints of typical floors were displaced 1 in., as in Fig. 6(*b*), resulting in the girder-shear corrections of 8.55 and 17.1 kips, shown. No wind was assumed to be acting on the bent in this study. Nevertheless, the bent deflected to the right, as shown, to develop corrective girder shears in the 1 : 2 ratio necessary to satisfy statics. It is to be noted that the effect of the unit displacement is to build up wind, direct, stress in the interior columns, which is assumed to be zero in the portal designing.

Complete analysis of the bent was made from data of this kind, the direct deformation of both interior and exterior columns being considered, and story-to-story distributions made until all displacements had been reconciled. Fig. 6(*c*) shows the resulting true girder shears in comparison with the portal shears used in designing. The corrections are negligible at the top and base of the bent; but at the fortieth floor (about two-thirds of the way up) the true shear in the interior girder is almost twice that in the exterior girders, instead of being equal, as would be assumed in a design by the portal method. Obviously, more strength is needed in the center panel. These results are parallel to Professor Witmer's qualitative predictions with regard to secondary stresses. However, the method of K-percentages (uncorrected) will yield a set of girder shears that follows closely the diagonal straight line of Fig. 6(*c*)—that is, the portal shears used in designing. Both these methods neglect column direct deformation.

Since the ratio of the true girder shear in the center panel to that of the outside panel differs with every floor (as is apparent from Fig. 6(*c*)), it follows that the contribution of a floor-to-wind column, direct, stress, or vertical reaction, will not be in the same ratio at every floor. This is illustrated graphically in Fig. 7, in which, in Tier (*a*), the contributions of each floor to column direct wind stress have been reduced to column unit stress, plotted to the vertical scale shown, and marked "increments." At the first floor the increments are such that there is none for the inside column—that is, portal girder shears exist there. However, at practically all other levels, the diagrams differ widely from the portal shape. At the twentieth floor, the Spurr relationship (that is, the straight-line variation of column unit stress) exists.

Tier (*b*) shows the accumulated, column, direct, unit wind stress at several levels. It was obtained by a summation downward of the previous data of Tier (*a*) and therefore consists of column "vertical reactions," as marked. The shapes of the various diagrams of Tier (*b*) are rather variable. None of them has the zero value of inside column stress assumed by the portal method used in designing the bent.

The diagram of "vertical reactions" at the base of the bent, at the bottom of Fig. 7, is worthy of note. Its shape may be thought of as a grand average of the performance of the bent at all other levels and, in this case, is very close to straight-line, Spurr, cantilever performance, instead of the portal distribution used in designing. This change-over is due to the direct deformation of the columns, and suggests that this bent could have been designed as well by the Spurr method from the first. The tendency for the inside columns

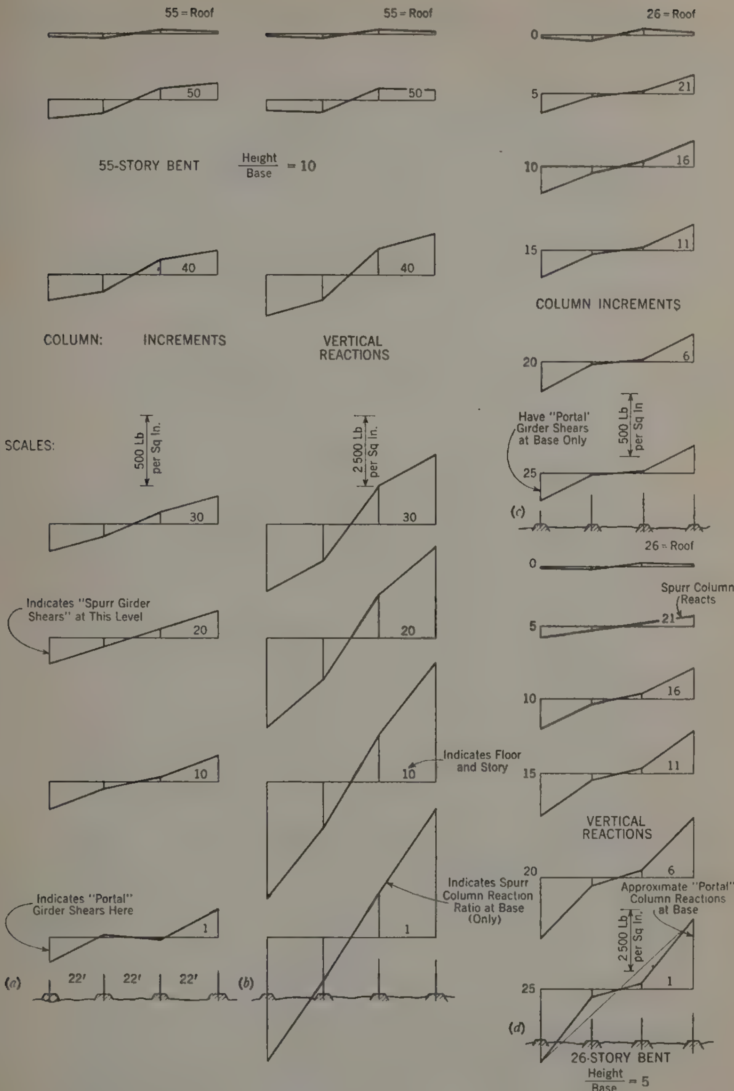


Fig. 7.—Precise Calculated Performance of "Portal-Designed" Bents (Direct Deformation of Columns Considered)

to participate in wind, direct, stress is quite pronounced in the upper half of the bent (Fig. 7(*b*)), where the accumulation (upward) of outside column, direct deformation, is great, thus bringing the inside column into action.

Results for a Twenty-Six-Story Bent.—At the right side of Fig. 7, Tiers (*c*) and (*d*) show corresponding diagrams for an exactly similar bent, except that it was only half as high, the height-to-base ratio being 5. The effect of column, direct deformation was not nearly so pronounced in this case. The greatest increase in true girder shear beyond the portal-calculated value was only 13%, instead of the 41% noted for the more slender bent. Tier (*d*) indicates little direct stress in these inside columns. Thus, the performance of the bent is substantially "portal," as designed; and, the effect of column, direct deformation is scarcely appreciable in a bent of these proportions.

(*C*) Torsional Effects of Wind on Buildings

General.—Through the courtesy of Professor Large, the Sub-Committee is able to present the results of his preliminary study of the torsional effects of wind on buildings.

In designing tall building frames it has been customary to assume the pressure of the wind as uniformly distributed over the face presented to the wind. J. Charles Rathbun,[12] M. Am. Soc. C. E., has shown that the wind pressure is not even uniformly distributed across each story. As a consequence, the center of pressure is eccentric to the axis of a symmetrical tower, as shown in Fig. 8(*a*). The result is a torsional moment upon the tower which twists it and causes unsymmetrical stress distributions.

Fig. 8(*b*) shows that, even when the wind pressure acts symmetrically over the face of a building, if the building is unsymmetrical in plan about an axis parallel to the direction of the wind, the eccentricity of the center of rotation of the building with respect to the resultant wind force will cause twist, and unsymmetrical stress distribution. If the structure is unsymmetrical about both north-south and east-west axes, as in Fig. 8(*c*), a wind from any direction will twist it. This is the type of structure hereafter considered at some length, under the convenient assumption of uniformly distributed wind.

Much credit is due Albert Smith, M. Am. Soc. C. E., whose work[13, 14] served as the groundwork for the study by Professor Large. A survey of engineering literature has revealed only one other reference[15] comparable to these.

The Center of Rotation of Towers.—For symmetrical towers the center of rotation is at the geometric axis of the tower, provided the framing also is symmetrical. For buildings of unsymmetrical floor plan the center of rotation is at the center of gravity (centroid) of the stiffnesses, or resistance values, of all wind bents in both directions.[13] In Fig. 8(*d*), the torsional moment $H\,e$ is

[12] "Wind Forces on a Tall Building," by J. Charles Rathbun, Proceedings, Am. Soc. C. E., September, 1938, p. 1335.

[13] "Wind Bracing," *Journal*, Western Society of Engineers (Chicago), February, 1933, p. 1.

[14] "Basis of Design for Hurricane Exposure," Report of Committee 308, by Albert Smith, author-chairman, *Proceedings*, Am. Concrete Inst., Vol. XXVII, 1931, p. 903.

[15] "Continuity in Concrete Building Frames," Second Edition (1937), published by the Portland Cement Association, p. 50.

resisted by the four east-west bents and the three north-south bents. The stiffness, K, of a wind bent may be defined as the load required to deflect it laterally a unit distance.

Analysis Procedure.—In analyzing a bent the wind pressure is resolved into a normal force H at the centroid of pressure, and a torsional moment $H\,e$ about the center of rotation of the bent. In Fig. 8(d) the four east-west bents will share the normal force H in the ratio of their stiffnesses, K, all bents thus deflecting eastward the same distance, under normal wind force only.

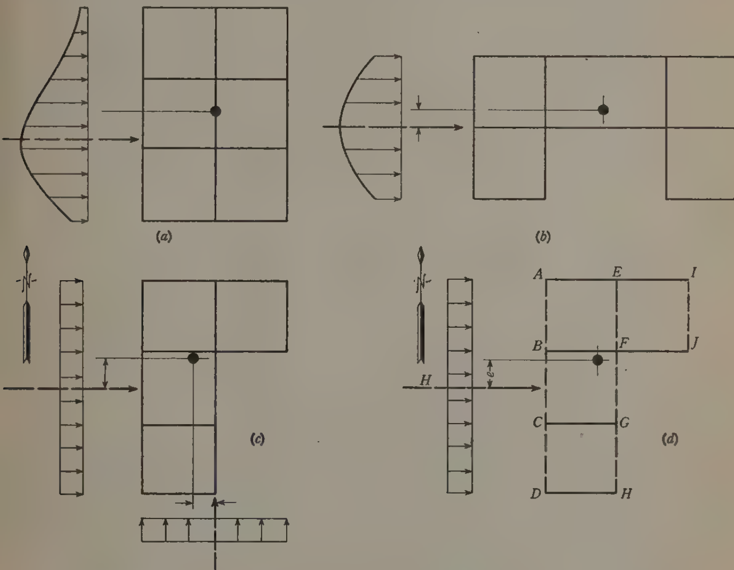


FIG. 8.—TORSIONAL EFFECT OF WIND

In addition, due to the torsional moment, each of the seven bents of the tower will be stressed in proportion to its distance from the center of rotation, and also in proportion to its K-value in accordance with the expression, $\frac{Mc}{I}$, often applied to rivet groups, giving

$$H_t = \frac{K\,H\,e\,c}{\Sigma(K\,d^2)} \qquad (6)$$

in which H_t is the wind shear induced in each bent due to torsion, c is the distance of that bent from the center of rotation, K is its stiffness, and $\Sigma(K\,d^2)$ is the sum of the products of the K-value of each bent times the square of its distance from the center of rotation.

An examination of all the foregoing procedure will reveal that the accuracy of an analysis will depend upon the K-values used throughout.

Analysis of Twenty-Six-Story, Unsymmetrical Tower.—The structure shown in plan in Fig. 9 was assumed for study because it had the minimum number of members required for dissymmetry about both axes; and also because only two of the seven bents, *A B C D* and *E F G H*, had interior columns whose

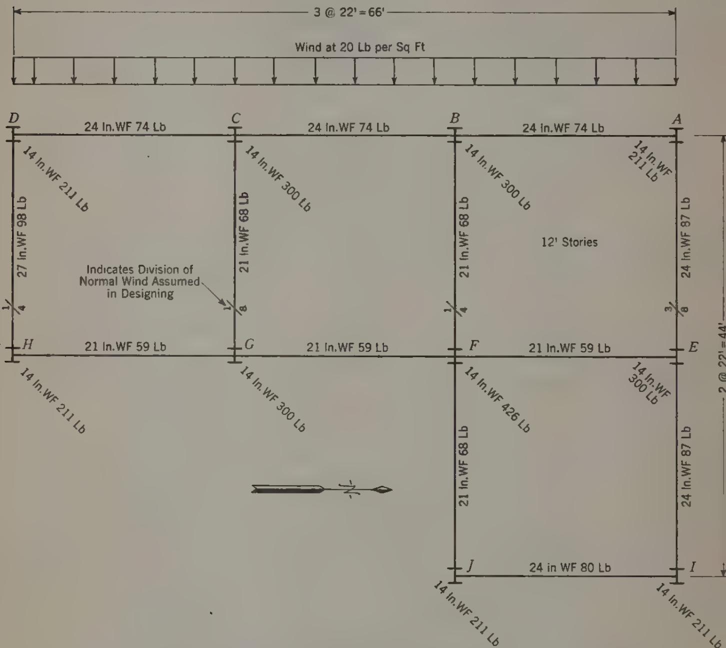


FIG. 9.—COLUMN AND GIRDER SECTIONS AT FIRST FLOOR OF TWENTY-SIX-STORY EXPERIMENTAL BUILDING FRAME

direct deformation might contribute to warpage of the floors under normal wind load. This was considered advantageous later for studying the warpage due to the torsional moment alone. Members had been proportioned quite casually for strength only, following the specifications of the American Institute of Steel Construction. Columns had been chosen using tributary floor areas, and considering the spandrel load equivalent to 6 ft of floor; and, the girders had sufficient strength to resist a normal wind of 20 lb per sq ft in either direction, assuming no torsion. The division of the normal wind load (in pounds per square foot) assumed in selecting the girders was as follows:

Description	Girders	Columns
Dead Load	100	115
Live Load	50	30*
Total	150	145

* Reduced

Sections shown in Fig. 9 are at the floor studied, twenty-five stories down from the top.

Methods of Computing K-Values.—Without speculating on the deformation of connections or the resistance of the architectural clothing of the building, at least four different methods may be used to calculate the relative deflection of the wind bents, depending upon what deformations are taken into consideration. These are:

(1) Bending of columns and girders and direct deformation of columns;
(2) Bending of columns and girders only (Smith's method);
(3) Bending of girders, and direct deformation of columns; and,
(4) Bending of girders only.

Table 3 is a comparison of results obtained by using these four methods to determine the division of the normal wind load among the east-west bents.

TABLE 3.—DIVISION OF WIND FORCE AMONG BENTS; FRAME PLAN SHOWN IN FIGURE 9

Method	Deformations Considered		Division of Normal Wind Among East-West Bents (Percentages)				Remarks
	Bending	Direct	*A E I*	*B F J*	*C G*	*D H*	
(1)	Column + girder	+ chord	38.2	28.8	13.3	19.7	Most precise
(2)	Column + girder	....	37.0	25.2	12.9	24.9	Smith's method*
(3)	Girder	+ chord	40.0	27.6	12.4	20.0	Sufficiently precise
(4)	Girder only		38.4	23.3	11.5	26.8	Most rapid

* Correction for chord deflection made subsequently.

It was found in using Method (1) that the bending of girders was responsible for most of the deflection of the bents, and that the bending of columns contributed the least in every case. The additional consideration of the direct deformation of columns (called chord deflection), increased the calculated girder-plus-column bending deflection by a minimum of 10% in Bent *B F J* (Fig. 9), and a maximum of 60% in the (narrow) Bent *D H*, and demonstrated that chord deflection is an appreciable factor in evaluating stiffnesses in a building of these proportions.

Presumably, Method (1) is the most accurate, and so the other methods indicated in Table 3 may be judged by comparing them with it as a standard. Note that Mr. Smith's method gives substantially the same results as Method (1), except in the narrow wall Bent *D H*, where the neglect of a relatively large chord deflection results in the assignment of 24.9% of the load to Bent *D H*, instead of 19.7 per cent.

Method (3) gives noticeably better results than Method (2) and entails about the same amount of work. (The computation of chord deflection has been demonstrated by Messrs. Large, Carpenter, and Morris.[16] The lateral deflection at the top of a wind bent, due to direct deformation in the column only, is reduced to the formula

$$\Delta_c = \frac{2\, s\, h^2}{3\, E\, L} \qquad (7)$$

[16] *Bulletin No. 93*, Ohio State Univ. Eng. Experiment Station, Columbus, Ohio, p. 23.

in which—adapting the notation of this report: h is the height of the bent; s = the direct unit wind stress in the outside column at the base of the bent; and, L = the width of the bent.)

In Method (4) one merely adds girder stiffnesses, $\frac{I}{L}$, across each bent and divides the total wind load proportionately. It is unexcelled for quick estimates, and sufficiently accurate in most cases. This method was used to check, quickly, the wind load on 9 east-west bents of an L-shaped building. The greatest deviation of these results from the Method (2) loads cited[15] was 3 per cent.

For unsymmetrical buildings with extreme variations in the width of bents Professor Large prefers to include the effect of chord deflection, as in Method (3). Chord deflection is increased four times when the base width of a bent is reduced by one-half. In the present report only one analysis of the experimental structure can be shown, and it has been performed using Mr. Smith's method (Method (2), Table 3).

Table 4, with Fig. 10, presents an analysis of the structure under east-west wind and torsion. In the first four columns the normal wind was divided

TABLE 4.—Wind Stress Analysis of Unsymmetrical Building

Bent	Stiffness, K	Division of Normal East-West Wind, H		Distance, d, from center of rotation, in feet	$\Sigma(Kd^2)$	Loads, in Kips				Remarks concerning girders
		Percentages	Kips			Torsional wind load, H_t	Total load, H, per bent	Computed Portal Wind Shears in Girders		
								Total	Safe	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(a) East-West Stresses; Primary										
A E I	0.524	37.0	149.5	27.65	57 700	23.8(W)	125.7(E)	34.2	41.5	24 in. WF 87 lb*
B F J	0.356	25.2	101.8	·5.65	1 640	3.3(W)	98.5(E)	26.9	27.6	21 in. WF 68 lb*
C G	0.183	12.9	52.2	16.35	7 040	4.9(E)	57.1(E)	31.2	28.2	18 in. WF 85 lb
D H	0.352	24.9	100.5	38.35	74 500	22.2(E)	122.7(E)	66.8	53.2	21 in. WF 142 lb
Total	1.415	....	....	....	....	Σ = 0	404.0	....	....	....
(b) North-South Stresses; Secondary										
A B C D	0.594	0	0	15.2	19 730	14.8(S)	14.8(S)	2.7	27.0	24 in. WF 74 lb*
E F G H	0.394	0	0	6.8	2 620	4.4(N)	4.4(N)	0.8	16.6	18 in. WF 77 lb†
I J	0.220	0	0	28.8	26 200	10.4(N)	10.4(N)	5.7	30.4	21 in. WF 103 lb†
Total	....	....	....	$\Sigma(Kd^2)$ = 189 430		Σ = 0	....	....	....	....

* Stands. † Required under a north-south wind.

among the four primary bents in proportion to stiffness factors computed from the contribution of column and girder bending to composite deflection. The

value of K in Column (2), Table 4, was computed by the formula

$$K = \frac{1}{\sum \frac{d^2}{I_c} + \frac{h}{n\,B} \sum \frac{b^2}{I_g}} \quad \text{.....................(8)}$$

in which (in the notation of the report): n = number of panels (not necessarily

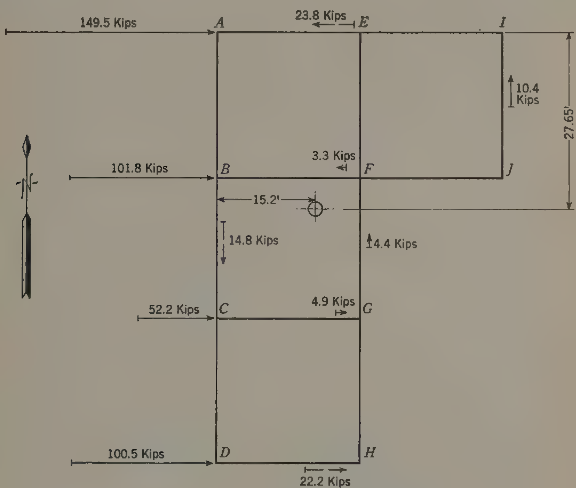


FIG. 10.—STRUCTURE ANALYZED IN TABLE 4

of equal length); B = width of a bent; and b = clear half-length of girders. To find the torsional center of rotation take moments about the top, as follows:

$$\begin{array}{rcr}
0.524 \times 0 & = & 0 \\
0.356 \times 22 & = & 7.80 \\
0.183 \times 44 & = & 8.05 \\
\underline{0.352} \times 66 & = & \underline{23.30} \\
1.415 & & 39.15
\end{array}$$

$$\frac{39.15}{1.415} = 27.65 \text{ ft}$$

The torsional moment of inertia of the whole structure, $\Sigma(K\,d^2)$, was computed next in Column (6) and used in the torsion formula

$$H_t = \frac{K\,H\,e\,c}{\Sigma(K\,d^2)} \quad \text{........................(9)}$$

to allocate the torsional wind shear, H_t, to each bent, in Column (7). The summation in Column (7) gave a check in each system of bents. The algebraic sum of Columns (4) and (7), Table 4, gave Column (8), the total shear in each

bent which, multiplied by the 12-ft story height and divided by the base width of each bent, gave girder wind shears according to the portal method of design, in Column (9). The safe girder shears, in Column (10), were computed from the girder sections, considering continuity from dead load plus live load.

Comparison of Columns (9) and (10), Table 4, reveals the fact that the girders of Bents *D H* and *C G*, Fig. 10, are deficient in strength, whereas those of Bents *A E I* are over-strong. Moreover, an examination will show that this condition is due to the torsion. Accordingly, in Column (11), stronger girders were selected for the two highly stressed bents. In doing this, care was taken that the original moments of inertia were maintained, thus preventing a redistribution of the wind to all the bents. Had sections with increased moments of inertia been selected, more wind would have been drawn to the bents being reinforced, thus largely defeating the purpose of the reinforcement.

The three north-and-south bents were only slightly stressed under the east-west loading. Under north-south loading, which is not shown, the situation was reversed, and the girders of Bents *E F G H* and *I J* had to be increased in strength to 18 in. WF 77 lb, and 21 in. WF 103 lb, respectively. No corresponding saving could be effected in Bent *A B C D*, where the total girder shear was only 81.5% of the safe value, assuming the necessity of meeting the moment of inertia requirement by using rolled sections.

Economy.—Having thus increased the tonnage of floor steel considerably, economy may be questioned and an alternative scheme considered. The Portland Cement Association[15] proposes to eliminate eccentricity and torsional moment entirely by making Bents *D H* and *C G* (Fig. 10) heavier until they are as stiff as Bents *A E I* and *B F J*, thus moving the torsional center to the line of action of the resultant wind force. A design so made, with one-fourth of the wind assigned to each of the four bents, effected an apparent saving in floor steel of 13% and diminished the tonnage to practically the original content necessary before torsion was considered. Such a showing is made possible only by neglecting the effect upon relative deflections of the direct deformation of columns under wind—that is, chord deflection. In order that the narrow Bent *D H* may have the same chord deflection as Bent *A E I*, under equal wind loads, the column areas of the narrow bent would have to be four times those of the wider bent. Moreover, since the chord deflection of Bent *D H*, as originally designed, is 60% of the column-plus-girder bending deflection, it is by no means a negligible factor. Obviously, over-all economy does not lie in the elimination of the torsional eccentricity except in lower structures where chord deflection may be negligible in comparison with bending deflection. Consequently, Professor Large advocates the computation of chord deflections in connection with the original division of wind load among the bents (Method (1) or Method (3), Table 3).

Column Stresses.—Fig. 11 is an isometric graph showing the accumulated chord stress at the base of each of the building columns. The shaded stacks show the wind, in kips, plotted with tension upward on the left and compression downward on the right. The dead load plus live load is also shown as a

compression. In Bent *D H* the wind exceeds the dead load, indicating uplift and transfer of stress to the adjacent bent, with attendant redistribution not heretofore anticipated. This is a typical tendency at the outer end of long narrow wings. It is believed that relief can be effected by: (*a*) The use of

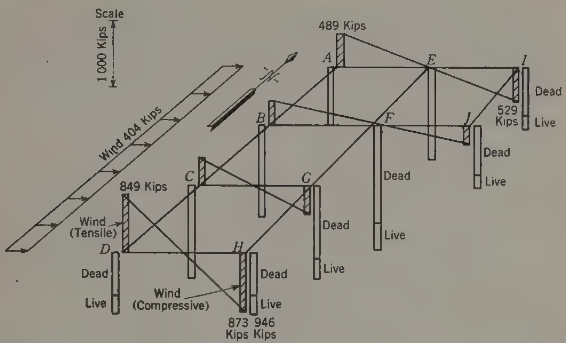


FIG. 11.—TOTAL COLUMN CHORD STRESSES AT BASE OF BUILDING, UNDER AN EAST-WEST WIND, DEAD LOAD AND LIVE LOAD

Method (3), Table 3, which distributes less wind to this bent; and (*b*) increasing the stiffness of Bent *C G*.

Effect of Accumulation of Column Direct Deformation.—In designing slender buildings and towers of symmetrical floor plan it is feasible to proportion the girder system for stiffness as well as for strength, as in the Spurr method,[6] so that when the columns extend and contract under wind, the floors will remain plane—that is, without warpage. On the other hand, if one is required to investigate an existing symmetrical structure that has not been proportioned for planar performance, the problem is not so simple.[11] As has already been shown in Part (*B*), it will be found that a warpage of the floors occurs and accumulates from the base of the structure upwards, causing secondary shear corrections in girders, which have maximum values approximately two-thirds of the way up the structure.

In the case of the unsymmetrical building under consideration, it will be found that if the wind-chord stresses shown in Fig. 11 are divided by the respective column areas, to give unit stresses, a rather pronounced floor warpage exists, particularly in the long wing. Mr. Smith has a "free-hand" method[13] of correction for following the effect of warpage upon the stresses in adjacent bents. Professor Large hopes to extend his own method of corrections[11] into this three-dimensional problem.

Features Requiring Further Investigation.—Doubtless, it is evident that the floor plan discussed (Fig. 10) is somewhat uneconomical from a wind-stress standpoint. Actual structures similar to it are not uncommon, and the profession should know how to place material where it will do the most good, and still not "load" the design. A re-design of the experimental structure which is predicated upon having no warpage of the floors is in progress. It is

believed, however, that a well-established method of following the stress effect of warpage, possibly that of Mr. Smith, would be of more practical value than idealistic devices to eliminate it entirely. Planar performance may not be an indispensible attribute of good design in all cases, even for symmetrical structures.

A study of service and maintenance records of buildings of the type discussed would be of incalculable value in correlating theory with actual performance. How much warpage will a floor system stand if necessary? What value of column wind stress should be tolerated if loosening of the masonry is to be avoided? Must one design the unsymmetrical building to have greater stiffness than the symmetrical tower? What evidence is there of the "twister" type of storm damage?

(*D*) Magnitude of the Assumed Wind Force on Tall Buildings

General.—In its first progress report the Sub-Committee proposed a tentative specification for wind force to be considered in the design of buildings.[17] This was a uniformly distributed force of 20 lb per sq ft for the first 500 ft of height, increasing from that level at the rate of 2 lb for each additional 100 ft.

Every effort has been made to elicit the views of qualified persons concerning a proper wind specification, with the result that many comments have been made in published discussions of the progress reports or in letters or unpublished memoranda received by the Sub-Committee. These have been studied carefully and the proposals herein contained have been based in part on that study and in part on the record of observations and investigations of the past seven years.

Arguments for Increase of Wind Force.—Although, in many cases, the suggested wind force was approved as adequate, or was characterized as somewhat too severe, objections were made to it on the ground that the requirements were not sufficiently exacting, particularly for the upper parts of tall buildings.

In a general manner the attitude of the critics of the specification was that designers do not know the variation of wind force with height for gales of extremely high velocities, and that they are still waiting for justification of present wind-pressure practice by experience gained with extraordinarily tall buildings in violent storms. Observed or estimated velocities in hurricanes or tornadoes were cited as justifying the adoption of a much more severe loading than that which was tentatively proposed in the first progress report.

D. C. Coyle, M. Am. Soc. C. E., stated[18] that there are no experimental data on the force of wind gusts for a duration of one or two seconds, which may be the critical duration for synchronization. Structural engineers need to know more about the length, width, depth, and violence of the gusts, particularly the size and periodicity.

Mr. Smith[19] regarded the recommended wind force as too small for gust effects at great heights. He cited, approvingly, Stanton's conclusion that

[17] *Civil Engineering*, March, 1931, p. 478.

[18] *Loc. cit.*, May, 1931, p. 700.

[19] *Loc. cit.*, May, 1931, p. 701.

gusts of practically uniform intensity may cover a large area at some distance from the ground. He suggested 40 lb per sq ft at any height above 500 ft, decreasing by steps to 15 lb per sq ft at ground level, as indicated in Curve 4, Fig. 12, thus giving for a 1 000-ft tower 56% more wind than would be comprehended by the Sub-Committee's loading (Curve 2, Fig. 12). Essentially the

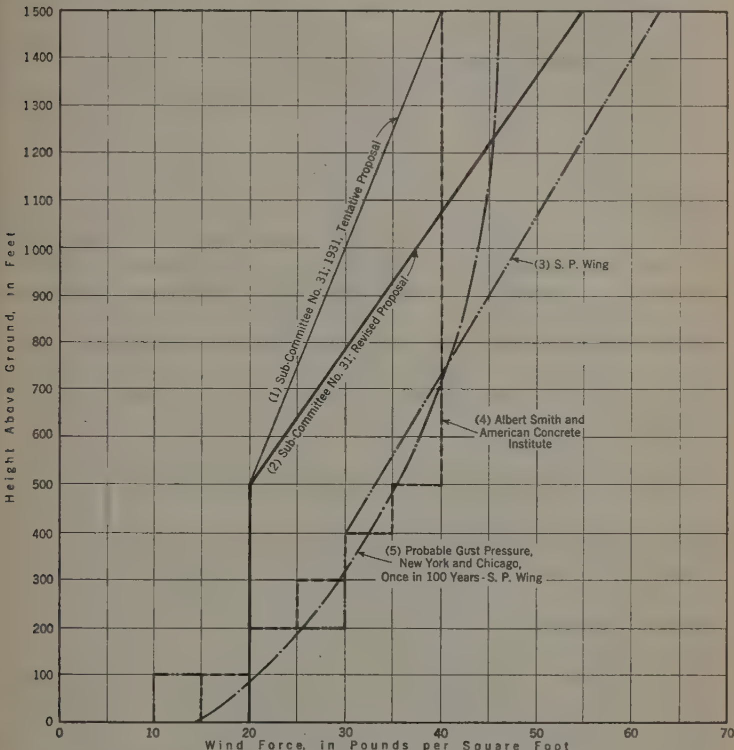


FIG. 12.—REVISED WIND FORCE PROPOSED FOR TALL BUILDINGS AND ITS RELATION TO OTHER PROPOSALS

same loading was recommended in the Report of Committee 308 of the American Concrete Institute on the "Basis of Design for Hurricane Exposure," of which Mr. Smith was author-chairman.[20]

Returning to the subject later,[21] Mr. Smith expressed the view that it is impossible to say how near their ultimate resistance some of the higher buildings with scant shelter have been, in major storms. Unless there is at least one

[20] *Journal*, Am. Concrete Inst., March, 1931, p. 903.

[21] *Proceedings*, Am. Soc. C. E., May, 1932, p. 949.

large tower in each city block, there can be no dependable shelter above the twenty-story level.

R. A. Philleo, Assoc. M. Am. Soc. C. E., considered that the prescribed wind force does not take into account the maximum observed velocities and wind pressures in the United States and Canada.[22] He cited the velocity of 128 miles per hr at Miami, Fla., and also Baier's computations of wind pressures in the St. Louis (Mo.) tornado of 1896, varying from 44.8 to 79 lb per sq ft, as well as the pressure of 110 lb per sq ft estimated by the committee reporting on the central Illinois tornado of 1925.

The late E. W. Stern, M. Am. Soc. C. E., urged that consideration be given to tornado and hurricane regions, referring particularly to the St. Louis tornado and to the Florida hurricane of 1926.[23]

S. P. Wing, M. Am. Soc. C. E., contended that designs should provide for the maximum wind that might occur once in 5 000 years, rather than the maximum wind on record.[24] Only fragmentary data are available on the maximum instantaneous velocity, and engineers should not overlook the probability of synchronization, particularly when the vibration period is relatively long. Mr. Wing considered the proposed loading at the 1 000-ft level as by no means generally conservative.

Amplifying his former observation, he later expressed[25] the view that structural security should not be allowed to depend on a secondary specification controlling rigidity. He submitted presumptive evidence that gusts are distributed over fairly wide areas, that little shielding of the upper parts of buildings will take place, and that synchronous vibration is quite probable. Based on frequency studies, Mr. Wing estimated that the pressure at a point 500 ft above the ground would, at least once in 100 years (or the lifetime of a relatively permanent building), exceed the Sub-Committee's recommendations by 50% in the case of St. Paul, Minn.; 75% in New York, N. Y., and Chicago, Ill.; 100% for Dallas, Tex.; 130% for Mobile, Ala.; and 250% for Point Reyes, Calif. He observed that there is a great tendency to forget the storms and disasters of a previous generation and cites disapprovingly the gradual reduction of prescribed wind force that has taken place after it had been fixed at 56 lb per sq ft for the design of the Forth Bridge while the Tay Bridge disaster was still fresh in mind. In further support of his contention Mr. Wing recalled that the committee on the Florida hurricane estimated the load on the complete structure as of the order of 50 lb per sq ft. Finally, he suggested for the New York area 10 lb per sq ft for the first 100 ft of height, 20 lb per sq ft for the next 100 ft, and 30 lb per sq ft for the next 200 ft. For buildings higher than 400 ft he would add 3 lb per sq ft for each additional 100 ft, as shown in Curve 4, Fig. 12. The relation of this specification to the probable gust pressure in New York and Chicago once in 100 years is also indicated (see Curve 5, Fig. 12).

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[22] *Civil Engineering*, June, 1931, p. 870.

[23] *Loc. cit.*, July, 1931, p. 951.

[24] *Loc. cit.*, p. 952.

[25] *Proceedings*, Am. Soc. C. E., August, 1932, p. 1103.

L. J. Mensch, M. Am. Soc. C. E., stated that it is well known that once in 25 or 50 years wind pressures may be much higher than those recommended by the Sub-Committee.[26] Mr. H. L. Dryden recalled that the experimental study of gusts by R. H. Sherlock, M. Am. Soc. C. E., and Mr. M. P. Stout showed that, in gusty wind, high velocities occasionally occurred over large areas.[27]

In an unpublished monograph, Mr. J. D. Marshall has recommended that, for the Kansas City (Mo.) area, provision should be made for tornadoes by requiring unshielded buildings to be designed for 30 lb per sq ft for the first 50 ft of height with an additional 1 lb per sq ft for each successive 10 ft in height. This would give 25 lb per sq ft at 500 ft and 125 lb per sq ft at 1 000 ft, above the ground.

Mr. Marshall urged strongly that the prescribed load be made greater for areas where tornadoes might be expected. He quoted approvingly the report of the Building Code Committee of the United States Department of Commerce, November 1, 1924, on "Minimum Live Loads for Use in Design of Buildings," recommending the use of a pressure of 60 lb per sq ft in determining the stability of buildings in localities subject to tornadoes.

Striking demonstration of the possibility of disastrously high wind velocities in even the northerly States was afforded on September 21, 1938, when a tropical hurricane moving northward on the regular hurricane path through the Western Atlantic encountered an area of high pressure and was diverted across New England and up into Canada. Reliable reports placed the sustained wind velocity at from 85 to 100 miles per hr, with gust velocities as high as 173 and 186 miles per hr.[28] The chances of the occurrence of wind gusts of 125 to 150 miles per hr anywhere in the Eastern States consequently cannot be ignored.

A useful indication of the probable intensity of the wind force on a tall tower in a metropolitan center is afforded by the investigation of the wind forces on the Empire State Building reported by Professor Rathbun.[12] Velocities as high as 102 miles per hr were noted. From a consideration of the observed deflections in relation to the behavior of a model of the building he computed that for a wind velocity of about 60 miles per hr the total wind force on the south face, including the suction effect, was 13.4 lb per sq ft.

If a hurricane with a gust intensity of 120 miles per hr (which appears to have been exceeded in the New England hurricane of September 21, 1938) were to strike the building, the probable wind force, assuming it to increase with the square of the velocity, would be $4 \times 13.4 = 53.6$ lb per sq ft. It is thus within the bounds of possibility that a wind force of more than 50 lb per sq ft might be encountered on the upper parts of unusually tall buildings in New York.

Revised Proposal of Sub-Committee.—Although it is impressed with the seriousness of the responsibility that must be assumed in recommending a

[26] *Proceedings*, Am. Soc. C. E., May, 1932, p. 944.

[27] *Loc. cit.*, p. 953.

[28] *Engineering News-Record*, September 29 and October 6, 1938; also, *Proceedings*, Am. Soc. C. E., January, 1939, p. 148.

design wind force for areas that may at some time be subjected to hurricanes or tornadoes, the Sub-Committee does not feel that it is practicable to require buildings generally to be proportioned for the extraordinarily heavy wind loads mentioned by Messrs. Philleo and Marshall. It believes that the best plan is to set up what might be regarded as a standard wind load for the United States and Canada with the provision that, in areas that are definitely known to be subject to hurricanes or tornadoes, a special wind specification be formulated locally. The intensity could then be made to conform to the results of special studies that have been made for these particular areas.

In recommending a general standard, consideration must be given to the probable exposure of the various classes of buildings affected by it. Especial difficulty and uncertainty arises in connection with very tall buildings in metropolitan centers. For these, it is probable that the presence of neighboring buildings of considerable height may shield the lower stories greatly and that the prescribed wind force may consequently be relatively small near the ground.

Having regard to all these circumstances, the Sub-Committee would recommend the following for the wind load on vertical surfaces:

(1) For the first 500 ft above the ground, a uniformly distributed force of 20 lb per sq ft;

(2) For that part of a building above the 500-ft level a force of 20 lb per sq ft, plus 3.5 lb per sq ft for each additional 100 ft of height; and,

(3) If the building is shielded or protected permanently, the wind force may be neglected for that part of the height so shielded, but not for more than the first 100 ft of the height.

The relation of the wind force suggested in Recommendations (1) and (2) to certain other proposals, and to the tentative wind force proposed in the First Progress Report, is shown in Fig. 12.

Respectfully submitted,

Clyde T. Morris,
N. A. Richards,
Francis P. Witmer,
C. R. Young, *Chairman*

January 19, 1939.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

LABORATORY INVESTIGATION OF FLUME TRACTION AND TRANSPORTATION

Discussion

By Y. L. Chang, Esq.

Y. L. Chang,[73] Esq. (by letter).[73a]—The valuable criticisms and ingenious contributions to the discussion have been of great benefit. Captain Kramer does not agree on the hypothesis of the tractive force on the stream bed by a non-uniform flow. The writer believes, however, that the investigation of the influence of non-uniform flow (although it complicates the problem) is undoubtedly necessary in order to study the actual condition of debris transportation in natural water courses. In the derivation of the expression for the decrease of potential energy, the same correction has been made independently by Professors Huang, Straub, and Rouse, and a revised conclusion has been drawn regarding the tractive force due to an accelerating flow in a horizontal channel bottom. The writer derived these equations on the basis of certain assumptions and certain reasoning which were not given in detail in the paper.

First, the writer considers the acceleration due to a convergence of stream lines in a non-uniform flow. The hypothesis is shown diagrammatically in Fig. 26. Professor Bakhmeteff has stated[74] that "when the stream lines are substantially inclined toward the cross-sectional plane, the acceleration $o\,a$ may have a noticeable component $o\,a'$ in the cross-sectional plane, the effect of which again will be to modify the distribution of pressures as caused by gravity alone." In the writer's derivation of the expression of potential energy between the stretch dx a mean excessive pressure of $\frac{w\,dy}{2}$ exerted on the down-stream cross-sectional plane is included. Moreover, Professor Bakhmeteff definitely

Note.—The paper by Y. L. Chang, Esq., was published in November, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1938, by Hans Kramer, M. Am. Soc. C. E.; June, 1938, by Messrs. E. W. Lane, and Joe W. Johnson; September, 1938, by Messrs. J. E. Christiansen, and W. H. Huang; October, 1938, by Lorenz G. Straub, Assoc. M. Am. Soc. C. E.; and February, 1939, by Hunter Rouse, Assoc. M. Am. Soc. C. E.

[73] Associate Prof. of Civ. Eng., Tsing Hua Univ., Yunnanfu, Yunnan, China; and Cons. Engr. to the Provincial Government of Kweichow, Kweiyang, Kweichow, China.

[73a] Received by the Secretary April 10, 1939.

[74] "Hydraulics of Open Channels," by B. A. Bakhmeteff, M. Am. Soc. C. E., Engineering Societies Monographs, 1932, p. 29.

shows that "elementary hydromechanics teaches that the distribution of pressure in a cross-section of moving fluid will obey the hydrostatic law and will be affected solely by gravity, when, and only when, flow takes place in such a manner that the fluid filaments have no acceleration components in the plane of the cross-section." Therefore, in the case of accelerated flows, it is obviously unsound to conclude that throughout the depth at any section the potential energy is equal to that at the water surface.

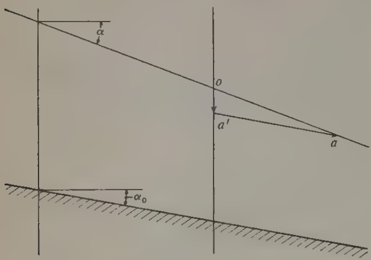

FIG. 26

Secondly, the experimental results of tractive force obtained in this particular flume are more likely consistent with the foregoing theory. However, this has been clarified by Professor Straub in his discussion that computation by Equation (17) gives values of tractive force that are too low, whereas neglecting the resistance due to the side-wall of the channel gives values that are too high. These two deviations have a more or less counterbalancing influence upon the result, so that the tractive force indicated in Table 3 seems to be in the proper order of magnitude.

In illustrating the effect of converging flow the following derivation is worth studying. Let $A\ B$ in Fig. 27 be a short stretch, within and near which the surface curve can be considered as a straight line with a slope α, and converges with the bottom at an angle $(\alpha - \alpha_0)$. The section that approximates the equipotential section at Point A most reasonably is an arc $A\ c$ struck with Line $A\ C$ as a radius. Call this arc length s; then,

$$s = r\,(\alpha - \alpha_0) \qquad \text{(102)}$$

in which r is the radius. The relationship between the vertical depth y and s could be formulated thus:

$$\overline{A\,c} = y \frac{\cos \alpha}{\cos\left(\dfrac{\alpha - \alpha_0}{2}\right)} \qquad \text{(103)}$$

$$r = \frac{\overline{A\,c}}{2} \frac{1}{\sin\left(\dfrac{\alpha - \alpha_0}{2}\right)} \qquad \text{(104)}$$

and,

$$s = \frac{y}{2} \frac{(\alpha - \alpha_0) \cos \alpha}{\sin\left(\dfrac{\alpha - \alpha_0}{2}\right) \cos\left(\dfrac{\alpha - \alpha_0}{2}\right)} = \frac{y\,(\alpha - \alpha_0) \cos \alpha}{\sin\,(\alpha - \alpha_0)} \qquad \text{(105)}$$

Equating the energy expressions:

$$\frac{w}{2\,g} [(V + dV)^2 - V^2] + \frac{T}{s}\,dx = w\,dx \sin \alpha \qquad \text{(106)}$$

As the volume considered is a unit with curved height s, the area subjected to tractive force should be $\frac{1}{s}$. From Equation (106) the expression of tractive force becomes

$$T = \left(- \frac{w\,V}{g}\frac{dV}{dx} + w \sin \alpha \right) s \ldots\ldots\ldots\ldots\ldots (107a)$$

or,

$$T = w\,(\alpha - \alpha_0) \left[y \frac{\cos \alpha \sin \alpha}{\sin\,(\alpha - \alpha_0)} = \frac{V^2}{g} \right] \ldots\ldots\ldots\ldots (107b)$$

For uniform flow, $\alpha = \alpha_0$, and

$$\operatorname*{Limit}_{\alpha=\alpha_0} T = w\,y \cos \alpha \sin \alpha \ldots\ldots\ldots\ldots\ldots\ldots (108a)$$

When α is small,

$$T = w\,y\,\alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (108b)$$

For non-uniform flow on a horizontal bottom, $\alpha_0 = 0$, and

$$T = w\,\alpha \left(y \cos \alpha - \frac{V^2}{g} \right) \ldots\ldots\ldots\ldots\ldots\ldots (109a)$$

When α is small,

$$T = w\,\alpha \left(y - \frac{V^2}{g} \right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (109b)$$

The writer believes that this derivation is more rational than the others previously given. As an arc of equal potential is considered there is no ac-

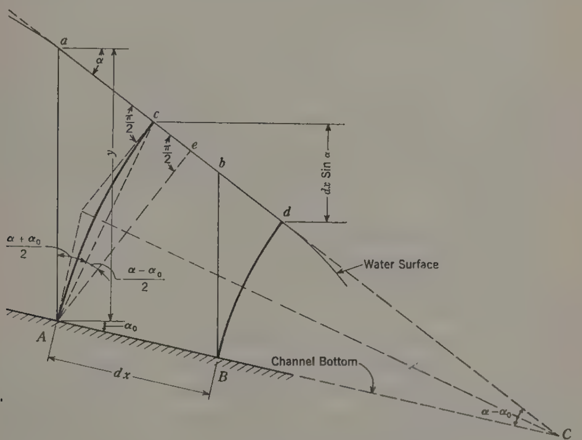

FIG. 27

celerating component in the directions normal to the stream lines. In Fig. 27 one observes that the potential drop of the water surface within the stretch dx, $dx \sin \alpha$, is evidently less than $dx \cos \alpha_0 \tan \alpha$, when two vertical depths $A\,a$ and $B\,b$, instead of two circular arcs $A\,c$ and $B\,d$, are taken for reference. This

supports the writer's original assumption that the difference of potential energy between the reach dx is less than that represented by the vertical drop from a to b. One can easily note, however, that the derivation is based on certain assumptions, such as:

(1) There is no excessive pressure due to convergence of flow on the arc $B\ d$;
(2) The distribution of static pressure is linear along the arcs;

One can also note that there is a question whether the tractive force should vanish when super-critical velocity occurs in a flume with a horizontal channel bottom. Unless assumptions of this nature could be proved and established definitely, one can scarcely venture to conclude that the foregoing expressions are the correct ones. For the time being, perhaps, it can be presumed that the derivation in the original paper, with due consideration for the convergence of flow, is possibly of the right order for high accelerating flows, whereas the revised expressions are more suitable for cases not much different from parallel flow.

Professor Lane takes a practical view of the situation pertaining to the bed-load movement in the Mississippi River. Accepting Equation (107*b*) as the correct expression and using Professor Lane's data, it can be shown that there will be motion upward on the sand slopes when the mean velocity of flow does not exceed $\sqrt{g\,y}\sqrt{\dfrac{\cos\alpha\sin\alpha}{\sin(\alpha-\alpha_0)}}$; or, numerically 1.44 ft per sec, above which sediments should move against the direction of flow. This only reveals one of the ambiguities encountered in these latter revisions. Actual records can be found in a report of the Mississippi River Commission.[75] The observation at Bullerton gives: $\alpha_0 = -\dfrac{4.4}{300} = -0.01466$; $\alpha = \dfrac{\alpha_0}{200} = 0.0000733$; $y = 10$ to 30 ft; and $V = 3.5$ ft per sec. The report also states[76] that "the quantity of material comprising sand waves was very small as compared with the quantity of material transported in suspension by the river. In the observations on the Fulton Section, Plum Point Reach, it was estimated that the quantity of material composing the sand wave was equal to not more than 2.5% of that carried in suspension."

In discussing sand dune progression the wave action of the water in the vicinity of the dunes should also be considered. A detailed investigation has been reported elsewhere.[77]

Regarding the question as to whether the silt load tends to decrease or increase the mean velocity of flow, the writer believes that the problem will be: Which is the more prominent factor—the viscosity of flow or the change of kinetic energy due to the suspension? The variation of the state of turbulence with the presence of silt in flow also might be one of the factors involved.

Mr. Johnson expresses regret that, with so many experiments and observations available, a general formula is still lacking that will express the

[75] "The Improvement of the Lower Mississippi River for Flood Control and Navigation"; prepared under the direction of T. H. Jackson, President, Mississippi River Commission, by D. O. Elliott, U. S. Corps of Engineers, Vol. 1, Table XXIII, St. Louis, Mo., 1932.

[76] *Loc. cit.*, p. 121.

[77] "Stauraumverlandung und Kolkabwehr," by A. Schoklitsch, Julius Springer, Wien, 1935.

quantity of debris movement. Painstakingly, he traces through a method of statical analysis to determine whether a particular formula offers distinct advantages over other formulas. He also reveals, correctly, the sources of error that are likely to affect the result in flume studies; but thus far, the application of these remedies in laboratory testing is not at all easy.

The part of the paper concerning the transportation of silt in suspension has been improved in several respects by Mr. Christiansen's discussion. He defines the validity of the suggested equations clearly. Other parts of the discussion, pertaining to the total load in suspension, and to the effect of silt on kinetic energy or on the Austausch coefficient of a stream, are also helpful. With regard to Samples 6 198 and 6 197, it is obvious that D is assumed equal to D_2, because of the flatness of the grains observed under the microscope.

Referring to Equation (33), the writer wishes to add the following equations so as to facilitate comparison with the other representative expressions reviewed under the heading "Published Authorities." Knowing that $T = 1.609\, w^2\, n^{6/5}\, s^{7/5}\, Q^{6/5}$, it is not difficult to convert Equation (33) into the following approximate form:

$$G = \frac{C\, n^{11/5}}{T_0^2}\, s^{7/5}\, Q^{3/5}\, (Q^{3/5} - Q_0^{3/5}) \quad \text{.....} (110)$$

in which $Q_0^{1.2}$ varies as $k_1 \frac{\rho_e\, D_0^{1/3}\, 2\,\beta}{n^{1.2}\, s^{1.4}\, M^{2\beta}}$. When ρ_e, n, and M are constant, and $\beta = \frac{1}{2}$,

$$Q_0^{1.2} = k_2 \frac{\rho_e\, D}{s^{1.4}} \quad \text{.....} (111)$$

in which C, k_1, and k_2 are constants.

In addition to the authorities cited in the paper, the writer wishes to include two more recent publications on debris transportation:

X.—H. J. Casey.—In the summary of his paper,[78] Casey gives two formulas for the rate of transportation, based on experiments with quartz sand (specific gravity, 2.65) with surface slopes ranging from $\frac{1}{400}$ to $\frac{1}{1\,200}$:

$$G = 0.367\, s^{9/8}\, (q - q_0) \quad \text{.....} (112a)$$

and

$$G = 0.333\, s\, (q - q_0) \quad \text{.....} (112b)$$

in which $q_0 = k \frac{D^{0.75}}{s^{1.25}}$, G and q are units of volume, and k is a constant. Equation (112a) is for uniform sand 0.5 mm $< D <$ 3.0 mm, whereas Equation (112b) is for mixtures.

XI.—A. Shields.—In his paper, Shields[79] attempts to connect the bed-load

[78] "Über Geschiebebewegung," by H. J. Casey, Mitteilungen der Preußischen Versuchsanstalt für Wasserbau und Schiffbau, Berlin, Heft 19, 1935.

[79] "Anwendung der Ähnlichkeitsmechanik und Turbulenzforschung auf die Geschiebebewegung," by A. Shields, Mitteilung der Preußischen Versuchsanstalt fur Wasserbau und Schiffbau, Berlin, Heft 26, 1936.

movement with turbulence and dynamic similarity. He obtains a dimensionless equation of the form:

$$\frac{G}{Q}\left(\frac{\sigma - \rho}{s\,\rho}\right) = 10\,\frac{T - T_0}{(\sigma - \rho)\,w\,D} \qquad (113)$$

Professor Rouse has discussed this work in some detail.

It may be interesting to demonstrate that there is a critical depth at which the mean load will be a maximum with all other hydraulic factors remaining unchanged. Using Equation (55) and equating to zero, the first differentiation of c_m with respect to d, one finds:

$$\alpha\,d\,e^{\alpha\,d} - e^{\alpha\,d} + 1 = 0 \qquad (114)$$

This condition could be satisfied only when $d = 0$. Therefore it may be assumed that the shallower the depth the heavier the concentration of silt in suspension will be, provided that other factors beside d remain unchanged.

Observations in the Yungtingho,[80] near Peiping, China, have made it possible to derive two empirical formulas for the total quantity of silt transported in suspension; one in terms of the flow discharge Q:

$$P = a\,Q^{1.54} \qquad (115a)$$

in which $a = 0.0095$ as an average (its maximum and minimum are, respectively, 0.0363 and 0.0026); and the other in terms of the flow velocity V:

$$P = b\,V^{3.89} \qquad (115b)$$

in which $b = 2.158$ as an average (its maximum and minimum are, respectively, 45.60 and 0.293). Both Q and V are in metric units and P is expressed as a percentage by weight. By combining Equations (115a) and (115b) one finds that V varies as $d^{0.656}$, which is not far from that given by Manning's formula.

[80] "A Complete Scheme for Controlling Yungtingho," Tientsin, China, Vol. 1, 1933, pp. 119–143.

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DISCUSSIONS

THE THREE-POINT PROBLEM IN A CO-ORDINATED FIELD

Discussion

BY R. ROBINSON ROWE, M. AM. SOC. C. E.

R. ROBINSON ROWE,[6] M. AM. SOC. C. E. (by letter).[6a]—The limited discussion of this paper may be summarized as: (1) Acceptance of the principles and formulas as correct; (2) statements of particular cases for which the methods are, or are not, adaptable; (3) doubt that the proposed method for the reduction of a four-point problem (or more) is economical; and (4) alternate forms for tabulating the solutions. The writer desires to express his appreciation of the thoughtful and helpful contributions of Messrs. Adams, Whitmore, and Jahn; with them, he is in substantial agreement.

Mr. Adams has stressed the increasing availability of co-ordinated spires in certain localities. His further comment on precision of co-ordinates would have been in point; but, regardless of original precision, spire co-ordinates cannot be considered valid until some check has been effected, due to deflection of structures. When so checked, however, accuracy sufficient for the next lower order of survey can be assured. If Mr. Adams implied the contrary in his final sentence, the writer cannot agree.

For example, the probable error of the location P_2 was not greater than 0.16 ft (0.13 + 0.03 ft), which is less than one-ten-thousandth part of the distance from the nearest spire and a much smaller part of the distance from the nearest instrument point of the prior triangulation. If another point, say 1 mile from P, were to be fixed in similar fashion, and if these two points were connected by traverse with satisfactory closure, all points of the survey should merit a fourth-order rating.

Spire co-ordination by the U. S. Coast and Geodetic Survey should be appreciated by surveyors because of their instant availability. Not only are instrument stations of the Survey often obliterated or covered by buildings,

NOTE.—This paper by R. Robinson Rowe, M. Am. Soc. C. E., was published in June, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1938, by Messrs. Oscar S. Adams, and George D. Whitmore; and December, 1938, by John R. Jahn, Assoc. M. Am. Soc. C. E.

[6] Associate Bridge Engr., State Dept. of Public Works, Sacramento, Calif.

[6a] Received by the Secretary May 1, 1939.

but residential construction on eminent hills frequently interferes with important sight lines.

The writer agrees with Mr. Whitmore that a five-point fix can be adjusted rapidly by the least squares method, if computers are familiar with the method and the five points are proved; but if one of the five points is rejectable, the method may introduce fallacy. Furthermore, if Mr. Average Surveyor should have occasion to adjust such a fix once in a year, or twice in a lifetime, he would probably select three points by "eeny-meeny-miny-mo," with one chance in ten of selecting the strongest figure. The semi-graphical method should appeal to him; admittedly, the writer's example would have been more clear if only five points had been studied.

Mr. Jahn has suggested an additional use for the three-point fix—the recovery of old instrument stations which had been referenced to several spires. Probably he refers to buried monuments, and for such objects the point is well taken. It was interesting to learn that he had also conceived of an instrument sight line as a band rather than the conventional geometric line; the width of the band should not be a "unit," but should be proportional to the probable error. His alternate computation forms and method of adjusting three-point and six-point fixes are worthy of consideration. Although the writer still prefers his own, he realizes that each computer will find certain arrays of figures more natural to his psychophysical processes. Although Mr. Jahn's six-point solution departs from the least squares solution by 0.11 ft (compared to 0.03 by the method of the paper), the lesser precision may be adequate and obtainable more economically.

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DISCUSSIONS

MOTOR TRANSPORTATION—A FORWARD VIEW A SYMPOSIUM

Discussion

BY THOMAS H. MACDONALD, ESQ.

THOMAS H. MACDONALD,[44] ESQ. (by letter).[44a]—The last paragraph of the paper emphasizes the fact "that the country has completed the pioneer stage of road development and every trend of highway development of the future must be an intelligent meeting of the particular service to be rendered." In order that there may be an intelligent meeting of the particular service to be rendered in the highway development of the future, it is essential that all pertinent factual information be available to highway engineers and administrators. In the paper reference was made to the State-wide Highway Planning Surveys. These are now (1939) being conducted co-operatively between the Bureau of Public Roads and 46 States and the Territory of Hawaii. Already these surveys have produced the factual basis for the solution of specific highway problems within a single State. They have also formed the basis for a comprehensive report from the U. S. Bureau of Public Roads, submitted to the Congress of the United States on April 27, 1939, by Special Message from the President of the United States. This report[45] was made in compliance with the specific direction of the Congress and consisted of two parts—Part I covering the "Feasibility of a System of Transcontinental Toll Roads," and Part II covering "Master Plans for Free Highway Development."

From the factual information developed in the State-wide Planning Surveys and other specific studies developed by the Bureau of Public Roads and the State highway departments, the definite conclusion was reached, and the recommendation made to the Congress, that it was not feasible to construct a system of transcontinental toll roads—three east and west, and three north

NOTE.—This Symposium was presented at the meeting of the Highway Division, Detroit, Mich., July 21, 1937, and published in June, 1938, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: September, 1938, by Messrs. F. Lavis, Edgar Dow Gilman, George Hartley, Robert Kingery, R. L. Morrison, and Roy F. Bessey; October, 1938, by Bruce D. Greenshields, Assoc. M. Am. Soc. C. E.; January, 1939, by Messrs. Robert B. Brooks, and H. George Altvater; and April, 1939, by W. W. Crosby, M. Am. Soc. C. E.

[44] Chf., U. S. Bureau of Public Roads, Washington, D. C.

[44a] Received by the Secretary May 18, 1939.

[45] H. R. Doc. No. 272, 76th Cong., 1st Session.

and south—with any expectation of such a system serving the genuine traffic needs of the United States, or of being self-liquidating. The facts and considerations leading up to this conclusion were fully discussed in the report.

Further analysis and study of the Planning Survey data clearly indicate the problems confronting highway engineers and administrators. Some of them are of long standing and have become acute through the tremendous increase in highway traffic. Not only increased numbers but also increase in normal speeds critically affects the various problems.

Much of the yearly obsolescence of the rural highways built in the past has been due to the restrictions imposed upon the design by inadequate rights of way. Another outstanding problem centers around the metropolitan areas and involves the provision of means for getting into, around, and through such areas without the delay that is involved in using present inadequate facilities. Traffic-flow maps indicate the natural flow lines between the various regions of the United States. A limited system of inter-regional highways approximating 1% of the total highway mileage constructed to modern standards is needed to provide adequately for the ever-growing traffic. Further study is needed to determine the location and approximate mileage in each State of such a system.

From a full discussion of these problems, there emerges the general outline of a master highway plan for the entire nation. The prosecution of the plan in all its parts calls for appropriate action by the Federal and State Governments and all county and municipal subdivisions. As desirable joint contributions of the Federal and State Governments, the report lists several undertakings as follows:

(1) The construction of a special, tentatively defined system of direct inter-regional highways, with all necessary connections through and around cities, designed to meet the requirements of the national defense in time of war and the needs of a growing peace-time traffic of longer range;

(2) The modernization of the Federal-aid highway system;

(3) The elimination of hazards at railroad grade crossings;

(4) An improvement of secondary and feeder roads, properly integrated with land-use programs;

(5) The creation of a Federal Land Authority empowered to acquire, hold, sell, and lease lands needed for public purposes and to acquire and sell excess lands for the purpose of recoupment.

The report emphasizes the difficulties encountered in the acquisition of adequate rights of way; and, in view of the fundamental necessity of such rights of way, proposes definite measures by which the United States could aid in the acquisition of suitable rights of way and simultaneously contribute helpfully to the solution of other urgent problems, especially certain problems confronting the larger cities.

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DISCUSSIONS

TRANSPORTATION OF SAND AND GRAVEL IN A FOUR-INCH PIPE

Discussion

BY G. W. HOWARD, JUN. AM. SOC. C. E.

G. W. HOWARD,[37] JUN. AM. SOC. C. E. (by letter).[37a]—It is desired to emphasize the fact that the data presented were obtained in connection with an investigation which was concerned primarily with methods for increasing the capacity of pipe lines transporting solids. The tests in question did not stress flow through plain pipe and, therefore, are not as complete as in the case of a study of plain pipe alone. The primary purpose was the development of the design of rifling referred to by Mr. Gladfelter.

Certain features of the apparatus have caused some concern. The most important of these are the method of introducing material into the tank and the sampling apparatus. Fig. 18, a general view of the apparatus, shows: (*a*) The line into the upper tank through which the material from the sump was pumped, (*b*) the upper tank, (*c*) the test line, (*d*) the trough for diverting the discharge, (*e*) the volumetric basin, and (*f*) the scales. Fig. 19 presents details of the sampling tube. It should be noted that this equipment was not used as a Pitot tube, but was simply an arrangement for taking representative samples from definite locations in the cross-section. Velocities at these various places were obtained volumetrically with this apparatus, the mixture being discharged into a standard graduate. As was originally stated, velocities determined in this manner were considered merely as weights "because it was not possible to obtain a satisfactory calibration of the sampling device." Average velocities in the pipe were greater than velocities obtained through the sampling device. The principal use of this sampling device was to determine solid concentrations at any given point in the cross-section. Average velocities in the test line were obtained by diverting the discharge from the

NOTE.—The paper by George W. Howard, Jun. Am. Soc. C. E., was published in September, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1938, by Messrs. Fred R. Brown, Joseph E. Montgomery, Elliott J. Dent, and David L. Neuman; January, 1939, by Morrough P. O'Brien, M. Am. Soc. C. E., and R. G. Folsom, Esq.; February, 1939, by Messrs. R. L. Vaughn, M. P. Durepaire, and Pierre F. Danel; and March, 1939, by H. S. Gladfelter, M. Am. Soc. C. E.

[37] Junior Engr., U. S. Waterways Experiment Station, Vicksburg, Miss.

[37a] Received by the Secretary April 17, 1939.

test line into a volumetric basin and weighing the trapped sample. The velocity in the test line was controlled by means of a positive-action valve which was located just below the upper tank.

FIG. 18.—GENERAL VIEW OF TESTING APPARATUS

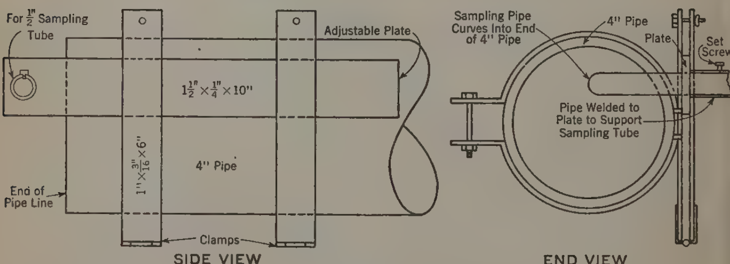


FIG. 19.—SAMPLING DEVICE

Of primary interest in the preparation of the paper was the gathering of data available on the subject of sand transportation in pipe lines. The additions made by Messrs. Brown, Durepaire, and Gladfelter tend to further this purpose. Messrs. Montgomery and Vaughn have shown that values of f are sensitive to changes in area; thereby, they clarified a misleading explanation of a phenomenon to which insufficient space had been given. Thanks are due to these gentlemen for clearing this point.

Colonel Dent prefers the Chézy formula to that of Darcy. This seems to be a matter of personal preference, because the value of C would of necessity become also a function of "coarseness of material, percentage of material in the mixture, and specific gravity of the mixture," which is objectionable to Colonel Dent but unavoidable under the existing circumstances.

Equation (10), suggested by Colonel Dent, is of particular interest. It is gratifying to find that the data which were presented were of value in providing an additional check on his formula. A compilation of Factor A would be of considerable value to all those engaged in the study of transportation of material in pipe lines.

Conclusions presented by Professors O'Brien and Folsom should not be considered at variance with those advanced in the original paper, because of the different range of velocities used for the two series of tests (velocities for the California tests were greater than for the writer's tests). Certain differences which seem apparent, possibly, may be reconciled as follows:

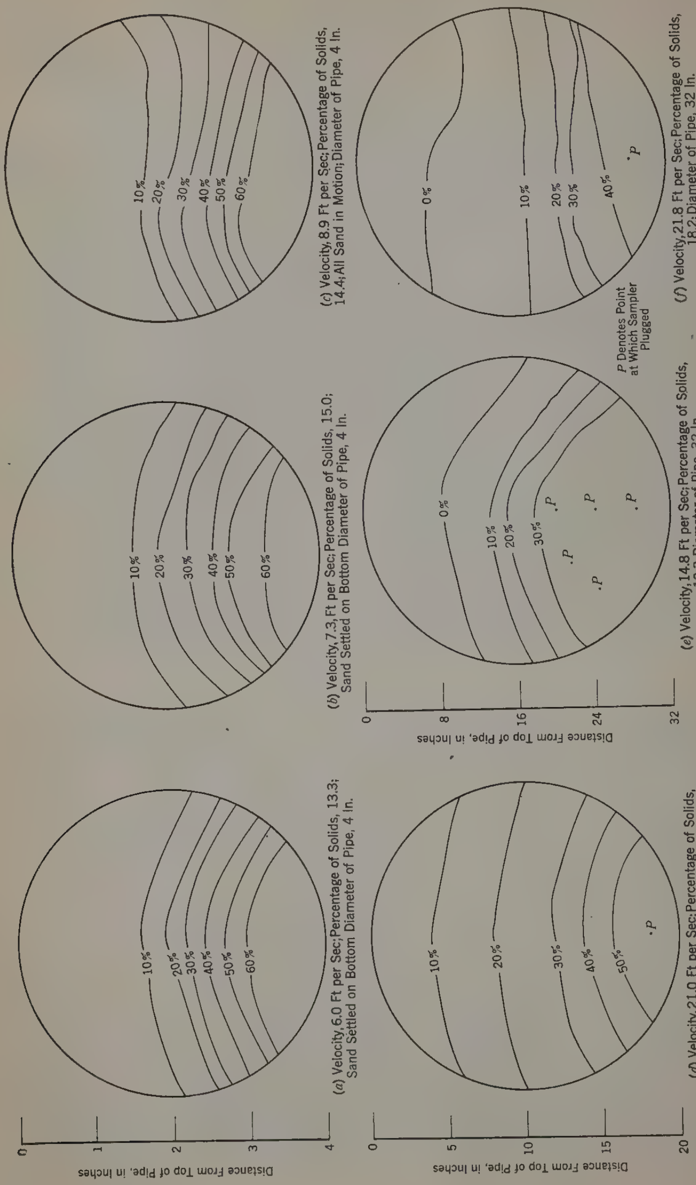
(1) The California tests indicated that, for a given sand concentration, there was a velocity below which sand tended to accumulate on the bottom of the pipe, ultimately causing the pipe to block off, if flow conditions were constant. The writer's tests on 4-in. pipe indicated that material was carried along the bottom of the pipe in the lower velocity range; but, if a certain concentration was exceeded for a given velocity, the pipe would block off.

(2) The California tests indicated that the head loss of the sand-water mixture was the same as for fresh water. The writer's tests on 4-in. pipe indicated that the value of the friction factor decreased with increases in velocity. Indications, from Table 1, are that the values of the friction factor tended to approach those for clear water.

Mr. Vaughn objects to "the apparent percentage of solids." This factor is admitted to have its limitations, but is one that is used with satisfaction in dredging practice on the Mississippi River. Furthermore, it is satisfactory for use in comparing flow characteristics of identical mixtures through two different types of pipe, as was the case for the study in question.

The implication that a general formula is impossible was not intended, as was interpreted by Mr. Vaughn. It was intended, however, to state that, from existing data, such a formula is not practicable. Colonel Dent shows that this formula may become an actuality in the future.

Mr. Vaughn indicates also that the correct nature of the problem has not been understood by investigators of the subject, and that the essential study is that of the flow of a mixture. It is granted that the study is that of the flow of a mixture, and this was realized all during the study. Mr. Vaughn, however, failed to offer any tangible means of remedying the testing procedure. For the purpose of the specific study the procedure was satisfactory and resulted in satisfactory conclusions. Criticism can be offered for all investigations after completion, but the value of the investigation lies in the application of the results. This particular investigation was for the purpose of proving that the "legend" referred to by Mr. Vaughn (that is, use of rifling



(*a*) Velocity, 6.0 Ft per Sec; Percentage of Solids, 13.3; Sand Settled on Bottom Diameter of Pipe, 4 In.

(*b*) Velocity, 7.3, Ft per Sec; Percentage of Solids, 15.0; Sand Settled on Bottom Diameter of Pipe, 4 In.

(*c*) Velocity, 8.9 Ft per Sec; Percentage of Solids, 14.4; All Sand in Motion; Diameter of Pipe, 4 In.

(*d*) Velocity, 21.0 Ft per Sec; Percentage of Solids, Unknown; Diameter of Pipe, 20 In.

(*e*) Velocity, 14.8 Ft per Sec; Percentage of Solids, 13.3; Diameter of Pipe, 32 In.

(*f*) Velocity, 21.8 Ft per Sec; Percentage of Solids, 18.2; Diameter of Pipe, 32 In.

FIG. 20.—DISTRIBUTION OF SOLIDS IN PLAIN PIPE

to increase capacities of dredge pipes) could be applied practically. The remarks of Mr. Gladfelter enlarge on the applicability of rifled pipe for dredges in the Mississippi River.

Although Mr. Durepaire feels that an "economical velocity" can be obtained with pump design, the use of this velocity as stated by Major Neuman "will not result in minimum unit total costs." This fact is particularly true in large dredging operations.

Mr. Danel has provided further interesting information in connection with the variations in head loss. The remarks by Mr. Gladfelter are quite pertinent and explain some of the reasons why the data originally reported upon were obtained in such a manner. The primary objective of the tests was the comparison of all types of transportation of materials in two pipes: One that was plain, and one that was rifled.

Distribution of material in a pipe is always of interest, and the data presented by Mr. Gladfelter add considerably to those already presented. Fig. 20 shows the distribution of material in 4-in., 20-in., and 32-in. pipe, prepared from material furnished by Mr. Gladfelter, and additional information obtained by the writer. The velocities in the various sizes of pipe were observed by different methods: Figs. 20(*a*), 20(*b*), and 20(*c*) were measured volumetrically; Fig. 20(*d*), by a velocity stick; and, Figs. 20(*e*) and 20(*f*) were measured by the salt-solution method.

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DISCUSSIONS

WATER-SOFTENING PLANT DESIGN

Discussion

By W. H. Knox, M. Am. Soc. C. E.

W. H. Knox,[13] M. Am. Soc. C. E. (by letter).[13a]—The seven discussions do not disagree with, or contradict, the writer's statements. Mainly, they amplify various items mentioned briefly in the paper, and thus give valuable additional information.

In an article of restricted length on such a broad subject only the salient features could be included. A textbook would have to be written to discuss details of all features of a water softening plant. Hence discussions could be continued indefinitely, since the design of such plants is changing continuously. Considerable time has elapsed since the paper was prepared and some minor items would be changed due to progress in the art.

The number of water softening plants is rapidly increasing, especially in the Middle West. If the present paper has helped in the understanding of the problems involved it will have served a useful purpose.

Note.—The paper by W. H. Knox, M. Am. Soc. C. E., was presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 21, 1937, and published in May, 1938, *Proceedings*. Discussion has appeared in *Proceedings*, as follows: September 1938, by Messrs. Philip Burgess, A. Elliott Kimberly, L. R. Howson, Charles P. Hoover, M. H. Klegerman, Rollin F. MacDowell, and D. E. Davis.

[13] Asst. Engr., State Dept. of Health, Columbus, Ohio.

[13a] Received by the Secretary May 17, 1939.

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DISCUSSIONS

PRINCIPLES APPLYING TO HIGHWAY ROAD-BEDS

Discussion

BY IRA B. MULLIS, ESQ.

IRA B. MULLIS,[35] ESQ. (by letter).[35a]—In this paper a new method is offered for designing road-beds. It follows well-known mechanical laws and the writer applies them in a manner similar to that followed by structural engineers.

The discussers have enhanced the value of many phases of the paper, and especially: (*a*) By contributions of basic data; (*b*) by pointing out certain structural relationships between road-beds and ceramic products; and (*c*) by the presentation and discussion of methods of design. The discussions will be commented upon topically and in the order of their appearance in *Proceedings*.

Method of Approach.—Professor Greaves-Walker states that the investigations are unquestionably along lines that will result in greatly improved road-beds and fills. He also states that in the manufacture of a good ceramic product, and in the making of a good road-bed, there is much common ground. In this connection he also stresses the value of ceramic principles which he believes should be applied to the structural design of road-beds.

Mr. Campen expresses approval of: (*a*) The analysis of the steps involved in consolidating water-soil mixtures; (*b*) the classification for road construction of a number of typical soil deposits; (*c*) the method used for showing the effect of water content on the structural properties of soils; and (*d*) the application of the information in the design of road-beds. Mr. Gray expresses his belief that the paper outlines the broader principles of road-beds in a concise and logical manner and recites his own experience in partial support thereof.

Mr. Turnbull says that the paper presents a clear picture of the functions of a soil in a road-bed, and that it reviews the various states of matter and the manner in which each one influences the action of a soil in the road-bed when placed at varying degrees of compaction. He also states that the definitions

NOTE.—The paper by Ira B. Mullis, Esq., was published in September, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1939, by Messrs. A. F. Greaves-Walker, H. Z. Schofield, W. H. Campen, Bernard E. Gray, W. J. Turnbull, and Bert Myers; and March, 1939, by E. Neil W. Lane, Jun. Am. Soc. C. E.

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[35a] Received by the Secretary May 22, 1939.

of the various characteristics of a soil, as well as the various types, are believed to be especially good. Mr. Myers says the paper is a valuable contribution to the literature on the subject of the design of road-beds and especially so because it is written in terms with which every engineer is already familiar. Mr. Lane states that the writer has raised several open questions and interpreted a great mass of fundamental data, but he does not express any view as to the validity of the interpretations.

Density, Porosity, and Moisture Content.—The density feature of the paper has been discussed at greater or less length by six contributors, and all agree that the strength of a road-bed increases with increasing density. This agreement is reached by a group of engineers representing a varied field of experience and research.

Professor Greaves-Walker says that there is no question about the structural properties of earth masses being enhanced and made more highly resistant to water penetration by increasing their density. Mr. Schofield says that Conclusions (1) and (2), relative to the effect of increasing density on the structural properties of earth masses and the resistance offered to the penetration of water, reached by the writer have been verified by Messrs. Everhart, Austin and Rueckel.[23] Researches by these gentlemen also show that the effect of de-airing clay bodies (and the consequent reduction of porosity and increase of density) increased plastic strength, the dry transverse strength and modulus of elasticity; and, in all but one case, it caused a marked increase in the water-slaking time of the dry specimens.

Mr. Campen states that resistance to pressure increases with increasing density of earth masses, and shows this relationship in Table 2. Mr. Myers contributes Table 4 which gives a detailed concept of the relationships of density, porosity, and water capacity of certain types of earth. In this table certain relationships appear to be common, if not universal. Of these, the following will be mentioned: (*a*) That the percentage of road-bed pore space filled with water during wet weather is generally high; (*b*) that the largest percentages of moisture are found in road-beds of low density (high porosity); (*c*) that the smallest percentages of moisture are found in road-beds of high density (low porosity); and (*d*) that the percentage of road-bed pore space filled with water appears to be reduced by the presence of considerable quantities of coarse matter, such as sand and gravel.

Mr. Turnbull is in accord with the belief that increasing soil density enhances its structural properties, increases its resistance to the penetration of water, and generally improves its desirability in all respects for road-bed use. However, Mr. Turnbull states that certain soils are known to have more detrimental swelling properties at the higher densities. The writer believes that such detrimental swelling occurs only in rare cases, none of which has come under his observation; and, he also believes that no type of earth can be compacted to an extent which impairs its resistance either to pressure or to weathering. The basis for this belief will now be exemplified by means of the inherent properties of a certain clay when compacted to given densities. The clay

[23] *Bulletin No. 74*, Ohio State Univ. Eng. Experiment Station, November, 1932.

selected is a gumbotil listed as Item No. 3, Table 4, by Mr. Myers. The discussion is as follows:

It is assumed that this gumbotil is to be placed in a road-bed in each of two states of compactness. It is also assumed that the magnitude of the compacting energy is ample for producing a density of 2.13 (Column (9)), and a water capacity of 10.3% (Column (16)). It is further assumed that the density required for Section *A* is 2.00, with a moisture content not in excess of 12%, and for Section *B* a density of 1.69, with a moisture content not in excess of 22 per cent.

Under these assumptions, the pore space in Section *A* was reduced so that its water capacity was only 1.7% in excess of that of its approximate shrinkage limit. Moreover, since the hygroscopic coefficient[36] of most clays is about 13%, this would aid in preventing further loss of moisture under field conditions. It will be seen, therefore, that the moisture range of earth in Section *A* is limited on the one hand by its hygroscopic coefficient and on the other hand by the restricted capacity of its pores. The capacity of the mass to shrink or swell, therefore, is reduced to the minimum, and its bearing capacity remains high as long as the mass is protected against weathering.

Section *B* was constructed with a water content of not more than 22% and hence, if it contained that quantity of water, was plastic at that time. Furthermore, regardless of the moisture content during construction, the pore space was such that the mass would become plastic whenever a sufficient quantity of water was available. Therefore, the pores, due to their large volume, would offer little resistance to the penetration of water. Assuming that a given drying would reduce the moisture content of the mass in Section *B* to 12%, which value is equivalent to that of Section *A*, the moisture loss since the time of construction would be 10%—a quantity ample for the development of shrinkage fissures of such size or number as would aid materially in the rapid distribution of water when available.

It may be seen, therefore, that the clay, consolidated as in Section *A*, is in a most favorable condition for resisting the loss of water either by evaporation or percolation. Moreover, since the restricted pore space is nearly filled at all times, the amplitude of moisture change is indeed small. By a similar process of reasoning, it may also be seen that the less compact mass of earth in Section *B* undergoes greater changes in volume and moisture content, and a more rapid loss of density when unprotected from the weather, than does Section *A*.

General Requirements for Density.—Within the decade prior to 1939, the necessity for developing considerable density in road-beds and earth dams had become increasingly apparent, and several standards of density have been set up. In 1932 the writer proposed the use of "standard" density as the ultimate density or ideal to be attained. In 1934 one State with which the writer was associated in Federal-Aid road work specified that road-beds must be compacted to 90% of "standard" density. Other standards of compaction have also become more or less popular, but the writer will discuss only his own unsatisfactory attempt to utilize "standard" density as a standard of compaction. Although this standard is an advance in improving the work, it was found to

[36] "Principles of Soil Technology," by Paul Emerson.

be unsatisfactory, inasmuch as the pore space at standard density bears no constant relationship to the quantity of water required for the production of critical softening or yield point due to water content. This lack of constancy of relationship may be exemplified by test values from Table 4 and shown in Table 6.

TABLE 6.—INCONSTANCY OF RELATIONSHIP BETWEEN THE LOWER PLASTIC LIMIT AND THE PERCENTAGES OF WATER REQUIRED TO FILL PORES IN MASSES AT STANDARD DENSITY (Item and Column Numbers Refer to Table 4)

Item No.	Column (12)	Column (16)	Ratio	Item No.	Column (12)	Column (16)	Ratio
1	14.9	14.3	0.96	17	74.0	58.4	0.79
3	22.0	10.3	0.47	19	20.0	17.0	0.85
5	17.0	14.7	0.86	31	19.2	20.4	1.06
8	15.7	19.5	1.24	42	13.9	20.5	1.47

Density Below the 2-Ft Level.—Of those who discussed this phase of the paper, the following comments were made: Mr. Campen states that the writer's suggestion that pore space in a road-bed at depths greater than 2 ft below subgrade should not exceed 40% of the volume is reasonable, except when applied to certain types of clay. He also states that locations are to be found where road-beds having a 50% pore space are entirely satisfactory for heavy loads because he believes water can be isolated from them. Mr. Myers states that an upper limit of 40% for pore space at depths of 2 ft below the subgrade seems to be entirely practicable and reasonable.

The writer's reason for suggesting an upper limit of 40% for pore space in a road-bed deeper than 2 ft below subgrade (when a lower percentage is preferable) is a two-fold one: (*a*) It is desirable to keep the cost of compaction as low as possible, but without sacrificing too much strength and durability; and (*b*) on a basis of an absolute density of about 2.65 for the vast majority of road-bed materials and the 40% pore space suggested in the paper, the bulk density would be 1.59 and the water required to fill the pore space would be 25.1% of the dry weight. Furthermore, since many earthy materials do not become plastic with less than 20% to 25% of water, the writer believes, on the basis of test data, that pore space in this part of the road-bed would commonly fail to become filled with water to an extent which would produce a plastic state. However, where conditions or materials are such as would produce instability, the upper limit of the 40% suggested should, of course, be lowered.

Density Above the 2-Ft Level.—Mr. Campen states that there is no doubt in his mind that, if all the soil or soil mixture courses within the upper 2 ft are compacted so as to restrict the water capacity to 70% of their plastic limit, the resistance required will be developed; but he also says that the pore space called for is lower and the thickness is higher than necessary and states his reasons for disagreement. Mr. Turnbull points out the difficulty, or even the impossibility, of compacting certain poorly graded sediments to densities suggested by the writer for use in the upper 2 ft of the road-bed.

He also says that it would seem to him that the factor of safety set for the upper 2 ft may be somewhat high; but he admits that, for certain soils, it may be entirely practicable and satisfactory.

Mr. Myers states that, in studies made under his supervision, on stabilized gravel base courses, which had a distribution of particle size and a degree of compaction such that the theoretical water capacity was below the limit suggested by the writer, these base courses did not become less dense or absorb additional moisture when exposed to the conditions existing on the road. He also states that samples taken from bases that contained 8% to 10% of moisture at the time they were constructed were found to contain only 3% to 7% of moisture two years later. Mr. Myers also conducted laboratory studies on the porosity-water content relationship through ten cycles of freezing and thawing, after which it was found that the specimens in which the theoretical water capacity was greater than 70% of the lower plastic limit of the material decreased in density and increased in moisture under the test procedure. Specimens with a theoretical water capacity of less than 70% of the lower plastic limit of the material did not decrease in density or gain materially in moisture content during the test.

Since no objection has been expressed relative to the 200-lb per sq in. load requirement proposed for road-beds, comments on this proposal will be omitted. It should be suggested, however, that the use of a sheeps-foot roller with a unit weight of not less than 200 lb per sq in. must be used for developing this strength. Furthermore, adequate compaction of earth demands that the unit weight of the roller must be increased with increasing plasticity. Moreover, the continued application of this unit weight to road-bed layers, until penetration ceases, would constitute an adequate test for compressive strength and uniformity of compaction.

The writer's reasons for suggesting a 2-ft thickness of special compaction for the uppermost part of the road-bed are as follows: (*a*) Road-beds in cuts are frequently composed of stratified, non-uniform or loose material which cannot be compacted adequately nor rendered water-tight without removal; (*b*) underground channels for water movement, if present, must be destroyed; (*c*) the uniformity of cut and fill sections of a road-bed should approach a constant density; (*d*) the cost of developing a given strength in a 2-ft thickness of road-bed by this method is commonly less than the cost of a 3-in. or 4-in. layer of granular material; and (*e*) road-bed compaction is a means of reducing the essential thickness of granular bases. A reduction in the thickness of this layer should be permitted only when this is warranted by the strength and water-tightness of underlying layers.

Mr. Campen and Mr. Turnbull have expressed doubt as to the necessity for restricting the pore space in the uppermost 2 ft of the road-bed to 70% of the lower plastic limit for the purpose of preventing critical softening. In reply to these objections, the writer would call attention to the authoritative definitions of the term "plasticity" recited by the writer under the heading "Solids." Moreover, the factor of safety suggested is only 70%, or as stated by Mr. Turnbull, 1.43. The writer had considered the advisability of suggesting the restriction of pore space to that of the lower plastic limit and de-

pending upon entrapped air content to act as the sole factor of safety; but subsequent experience demands a more adequate factor of safety. Tests have frequently shown that pores sometimes become 100% saturated, although the values are commonly lower (Table 4, Column (18)). Since engineers usually recognize the need of various means for compensating for small variations in the uniformity of structural materials, it would seem that a value of 1.43 is not too great, especially since a factor of safety of 2 is in common use for structural steel, which is quite uniform in quality as compared to most clays.

The inability to compact poorly graded materials to certain densities, as pointed out by Mr. Turnbull, was implied although not adequately presented in the original paper. Obviously, size gradation is an essential to the production of strength and resistance to the penetration of water. A comprehensive method for the predetermination of ideal grading for maximum density of earth mixtures has been described by Charles H. Lee, M. Am. Soc. C. E., in 1938.[37] Although Mr. Lee's work is based upon the well-known experiments on aggregate for concrete by Sanford E. Thompson and the late William B. Fuller, Members, Am. Soc. C. E., in 1903,[38] supplemented by those of A. N. Talbot, Past-President and Hon. M. Am. Soc. C. E., and F. E. Richart, M. Am. Soc. C. E., in 1919 to 1923,[39] he shows the application of this law to earth mixtures. Mr. Lee has also developed a splendid form of graphical diagram based on Professor Talbot's equation and includes five cycles, ranging in size from 0.001 to 152.4 mm.[40]

TABLE 7.—Effect of Plasticity Index on Resistance to Earth Failure

Plasticity index	Cementation (number of blows)	Absorption failure (time, in minutes)	Plasticity index	Cementation (number of blows)	Absorption failure (time, in minutes)	Plasticity index	Cementation (number of blows)	Absorption failure (time, in minutes)	Plasticity index	Cementation (number of blows)	Absorption failure (time, in minutes)	Plasticity index	Cementation (number of blows)	Absorption failure (time, in minutes)
36	200+	40	14	200+	30	8	200+	64	4	82	2	2	197	7
34	200+	5	14	200+	31	7	200+	120+	4	117	3	2	117	29
28	200+	120+	13	200+	49	7	200+	120+	4	129	7	2	136	60
31	200+	120+	13	200+	11	7	200+	120+	4	186	18	1	6	1
27	200+	60	13	200+	9	6	200+	120+	4	158	32	1	18	1
25	200+	5	12	200+	7	6	128	120+	3	158	120	1	22	2
22	200+	8	12	200+	120+	6	200+	33	3	162	2	1	71	2
20	200+	2	12	200+	120+	6	200+	12	2	43	1	0	6	1
16	200+	120	11	200+	53	5	200+	4	2	169	2	0	13	1
16	200+	120+	11	200+	53	5	146	83	2	65	3	0	12	1
16	200+	6	10	200+	120+	4	59	1	2	68	3	0	12	3
15	200+	20	9	200+	48	4	165	2	2	148	5	0	8	6

Plasticity Index-Strength Relationship.—Mr. Campen makes the statement that the load-bearing value at any given water content varies with the plasticity index. In partial support of this statement, it may be safely stated that highly plastic clays, such as gumbos, commonly show high strength, but china

[37] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), pp. 1–61.

[38] *Loc. cit.*, Vol. 59 (1907), p. 67.

[39] *Bulletin No. 137*, Eng. Experiment Station, Univ. of Illinois, October 15, 1923.

[40] *Proceedings*, Am. Soc. C. E., September, 1938, pp. 1438–1441.

and other clays of low plasticity are generally weak. The reader, however, should not infer that, because a certain clay shows a plasticity index of greater magnitude than that of a certain other clay, its cohesive strength is to be considered as being universally superior in resisting all types of forces. For the purpose of exemplifying any relationship of plasticity index to resistance of earth failure, Table 7 is shown. Cementation and absorption cylinders were 1 in. by 1 in. in size, molded under the usual pressures and dried under standard conditions.

The results of these tests indicate that, as a rule, the cylinders having low plasticity indices offered less resistance to blows and also to absorption of water than did those having high plasticity indices. However, it is also evident that the resistance to blows or slaking offered by the cylinders is not often indicated by the plasticity index.

Road-Bed Density and Pavement-Thickness Relationship.—Mr. Gray's contribution of design for "flexible type" pavements is sound in principle and his discussion is so practical and full of common sense that it should receive considerable attention. Moreover, his conclusions concerning the relationship between road-bed stability and pavement durability are quite apropos, regardless of the type of pavement laid.

In closing the discussion, the writer would stress the following essentials:

(*a*) Develop the road-bed material into the strongest and most water-resistant structure possible;

(*b*) Utilize selected local materials for enhancing the quality of the uppermost part of the road-bed; and,

(*c*) Complete the roadway with a pavement or wearing surface adequate for all requirements.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

WIND FORCES ON A TALL BUILDING

Discussion

BY COLIN SKINNER, ASSOC. M. AM. SOC. C. E.

COLIN SKINNER,[50] ASSOC. M. AM. SOC. C. E. (by letter).[50a]—In designing tall buildings numerous assumptions must be made in order to simplify the work to a practical degree. It is not rational, however, to go to great extremes in refinement of structural design unless the basic assumptions are substantially correct. Too much time and effort have been spent on devising methods of calculating wind stresses in tall buildings, based on the assumption that the steel frame carries all the stress, when it is obvious that the other materials increase both the strength and stiffness of a building. The data in Professor Rathbun's paper may not be complete, but it contains considerable useful information which should serve to stimulate a fresh approach, based on more logical assumptions. The American Institute of Steel Construction and the owners of the building should be congratulated for sponsoring this program and for permitting publication of the results.

In 1932 the writer made a series of observations on the various instruments installed in the Empire State Building for measuring wind pressure, deflection of the building, and stresses in the steel frame. A considerable number of observations were made over a 2-yr period, and many of these are included in Professor Rathbun's paper. In addition to these data, however, there are several additional observations which may help in interpretation of the published data.

For approximately six months after the building was completed, the strain gages on several columns immediately below the twenty-fifth floor showed no change in stress, regardless of the intensity of the wind. The plastered partitions surrounding the elevator shafts and public halls, the concrete floors, and

NOTE.—The paper by J. Charles Rathbun, M. Am. Soc. C. E., was published in September, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1938, by Messrs. David C. Coyle, and Clyde T. Morris; January, 1939, by Messrs. Robins Fleming, F. P. Shearwood, Lydik S. Jacobsen, Francis L. Castleman, Jr., J. B. Wilbur, R. D. Spellman, David A. Molitor, Walter J. Gray, and K. L. DeBlois; March, 1939, by Messrs. Albert Smith, and Victor R. Bergman; and May, 1939, by Otto Gottschalk, Esq.

[50] Architectural Engr., USHA, Washington, D. C.

[50a] Received by the Secretary April 17, 1939.

the exterior masonry walls stiffened the building to such an extent that practically no wind stress was transmitted to the steel frame.

First Big Storm.—Early in March, 1932, a severe storm hit the building with an indicated wind velocity averaging about 70 miles per hr, and gust velocities of more than 95 miles per hr. During this storm a series of muffled explosions were heard and some cracks developed in the masonry materials. At that time (1932) there were no tenants in the building between the twenty-fourth and the eightieth floors, with the exception of a few offices on the forty-first floor. The partitions surrounding the elevator shafts, public halls, and utility rooms (see Fig. 4) were all in place, however, and probably contributed to a large extent to the stiffness of the building.

A visual inspection made the day after this storm showed very few cracks in the partitions surrounding the elevator shafts. Diagonal cracks were observed in several office partitions perpendicular to the exterior wall on the forty-first floor, and in the partitions surrounding a motor room on the twenty-ninth floor. Cracks were also observed in the concrete floors at the first row of interior columns, parallel to the exterior walls.

These floor cracks seemed to indicate that the exterior columns were carrying a substantial portion of the direct stresses, in accordance with the portal theory. It should be noted, however, that the central core of the building contains deep knee-braces and many plastered partitions which undoubtedly restrained the floors from rotating in accordance with the cantilever theory. Furthermore, the floors were about a year old at the time of this storm, and the natural shrinkage of the concrete had probably produced internal tensile stresses which may have been close to the ultimate strength of the concrete. Many additional cracks have occurred in the floors since this storm as mentioned subsequently herein.

Groaning Period; Vibration.—On October 6, 1932, another severe storm occurred (Table 1, Item No. 82) with indicated wind velocities averaging about 70 miles per hr from the south southeast. During this storm the strain gages on the twenty-fourth-story columns moved back and forth very slightly, indicating that the steel frame was beginning to carry a small portion of the wind load. Groans could be heard as the vibration of the building apparently caused slipping between the steel frame and the masonry or grinding in the cracks, which had occurred in the first big storm. With each succeeding storm the groans caused by vibration decreased in intensity and the strain gages apparently became more sensitive, indicating that the building was becoming more flexible and that the steel frame was gradually assuming a larger percentage of the wind load.

After the storm of November 1, 1932 (Table 1, Item No. 102), it was noticed that winds of 30 to 35 miles per hr, against either of the broad faces of the building, would cause the strain gages to oscillate; but winds below this range caused no apparent change of stress in the steel. This indicated that the force of a 30- to 35-mile wind was required to overcome the internal friction and to cause the masonry to slip on the steel or along existing cracks. Thus, a severe storm would deflect the building several inches; then, as the wind subsided, the

deflection would decrease until the internal friction was equal to the restoring force of the deflected building. The building would then remain in this partly deflected position until the next severe storm.

It was also noted that the building was considerably stiffer in the longer direction. Before November 1, 1932, there was little deflection about the minor axis; but during this storm (Table 1, Item No. 103) additional cracking occurred, and the building began to deflect and vibrate appreciably about both axes. After this storm it was noticed that winds of 55 to 60 miles per hr against either of the narrow faces would cause the strain gages to oscillate, but winds below this critical range caused no apparent stress in the steel frame.

Existing Cracks.—In December, 1938, the writer made a visual inspection of existing cracks on several unoccupied floors, including the twenty-ninth, forty-second, and fifty-fifth floors. It was found that the cracks in the floors and walls were more numerous then than in 1933 (five years previous). Inspection of the floors showed prominent cracks extending between column centers in both directions. It is probable that these cracks were caused chiefly by natural shrinkage of the concrete, as they generally averaged about 0.12 in. in width with columns spaced about 20 ft on centers. It was noted, however, that in several places the floor on one side of a crack was more than $\frac{1}{16}$ in. above the level on the other side, and in some cases the maximum vertical displacement was nearly $\frac{1}{8}$ in. There were relatively few cracks in the plastered ceilings; but the plastered partitions surrounding the elevation shafts were seriously cracked in the lower portion of the main tower and cracked to a lesser degree in the upper stories.

Velocity of the Wind.—In studying Table 1, it should be noted that the New York City Observatory in Central Park is nearly 7.5 miles from the U. S. Weather Bureau. The anemometer in Central Park is 62 ft above the ground, but only about 30 ft above the trees. In addition, the park is entirely surrounded by tall buildings averaging approximately 140 ft in height. The U. S. Weather Bureau, near the southern tip of Manhattan, is 454 ft above the harbor, but is near the down-town "skyscraper" zone, and winds from the north, northeast, and east are seriously modified in velocity and direction by these tall buildings. The anemometer on the Empire State Building has a 32-blade rotor which is restrained by helical springs, attached by gears to a self-synchronous motor. When the wind blows the rotor turns through less than 360° and the angle of rotation measures the wind velocity. The anemometer on the Daily News Building is similar to the one on the Empire State Building, but is 30 ft above the roof and may be less affected by the up-draft on the windward side of the building.

Before January 1, 1928, the U. S. Weather Bureau used a four-cup anemometer which gave indicated wind velocities greater than the true velocity. During the next few years, however, a new three-cup anemometer was developed to give more accuracy and, since January 1, 1932, wind speeds published by the U. S. Weather Bureau have been corrected before publication to give true wind speeds. In analyzing wind records it is important to know whether the velocity represents the average for a 1-hr period, a 5-min period,

or a brief gust, as there may be considerable difference in these velocities. The velocities obtained at the Empire State Building are averages estimated after watching the anemometer dial for several minutes, but most of the other velocities are averages for a 1-hr period.

Wind-tunnel tests made at the National Bureau of Standards on a model of the Empire State Building indicate that the velocity 15 ft above the observation tower is about 23% greater than the velocity of the approaching wind. The wind velocities recorded in Column (4), Table 1, therefore, are about 23% too high and should be reduced when studying the actual wind pressures on the building. Another factor that should be considered is that during a severe storm the wind velocity 1 253 ft above the ground is undoubtedly greater than the velocity nearer the ground.

TABLE 7.—AVERAGE WIND VELOCITIES GROUPED BY INTENSITY

Observed velocity of wind, in miles per hour	Number of observations	Average Velocity of Wind, in Miles per Hour*: Empire State Building	Central Park Observatory	Daily News Building	U. S. Weather Bureau	Ratio of Velocities Compared with Empire State Building: Empire State Building	Central Park Observatory	Daily News Building	U. S. Weather Bureau
Less than 10	15	5.9	5.9	5.7	7.7	100	100	96	129
10 to 19	34	13.6	9.6	10.8	11.3	100	71	79	82
20 to 29	31	23.1	10.8	12.4	15.0	100	47	54	65
30 to 39	27	32.9	14.1	16.8	20.6	100	43	51	63
40 to 49	28	42.7	16.9	19.8	26.0	100	40	47	61
50 or more	28	59.2	21.0	26.6	29.8	100	36	46	51

* At Empire State Building wind velocities are averages for a brief period of observation, but at the other three observatories wind velocities are averages for a 1-hr period.

The winds in Table 1 were first arranged in six groups, according to velocity, and each group was averaged to obtain the data shown in Table 7. This study shows that the relative wind velocities at the four observatories change considerably as the storms increase in strength. With strong winds, low buildings receive considerable protection from surrounding buildings and the degree of protection is relatively greater during extremely severe storms.

TABLE 8.—RELATION BETWEEN HEIGHT AND VELOCITY

Location of anemometer	Height above ground, in feet	True velocity of wind, in miles per hour	Ratio
Empire State Building	1 253	84	100
U. S. Weather Bureau	444	71	85
Central Park Observatory	62	54	65

The relation between height and velocity may be estimated from the storm of March 22, 1936, at the top of Table 1, which gives an indicated wind velocity of 102 miles per hr at the Empire State Building. The true velocity of the approaching wind, therefore, was about 84 miles per hr, compared with 71 miles per hr at the U. S. Weather Bureau and 54.4 miles per hr at the New York City Observatory in Central Park (see Table 8).

On several occasions the writer has observed flags on various tall buildings flying in different directions with variations in direction as much as 90° at the same instant. In order to study the effect of surrounding buildings on velocity and direction, the winds of Table 1 were grouped by the direction indicated at the Empire State Building and the directions at the other observatories expressed as plus or minus degrees. This study showed fairly consistent differences for each particular direction and indicated that at the three lower stations both velocity and direction of the wind are seriously affected by the surrounding buildings.

On November 1, 1932, three sets of observations were made, as indicated by Items Nos. 100, 101, 102, Table 1. For several hours the wind was exceptionally steady with an indicated velocity of 75 miles per hr from the southeast. Just before noon, while the plumb-bob was being observed, the wind suddenly increased in intensity for a few minutes, swung around through south and then

TABLE 9.—AVERAGE WIND VELOCITIES, GROUPED BY DIRECTION

Number of observations	Direction of wind; Empire State Building	Velocities of Wind, in Miles per Hour				Number of observations	Direction of wind; Empire State Building	Velocities of Wind, in Miles per Hour			
		Empire State Building	Central Park Observatory	Daily News Building	U. S. Weather Bureau			Empire State Building	Central Park Observatory	Daily News Building	U. S. Weather Bureau
1	N	45	18	19	26	3	SW	35	14	16	15*
1	NNE	32	18	24	13*	6	WSW	36	15	17	22
5	NE	42	19	28	14*	6	W	35	14	17	24
5	ENE	58	25	34	12*	14	WNW	43	17	16	28
1	E	55	21	27	9*	13	NW	48	20	19	31
6	ESE	51	15	28	28	5	NNW	54	22	24	38
6	SSE	58	16	26	36						
4	S	35	13	18	21						
6	SSW	39	15	19	23	Total		666	262	332	277
						Average		43.4	17.5	22.1	27.7
						Ratio		100%	39%	50%	62%

* These points have been omitted from the average because tall buildings apparently shield the anemometer when winds are from these directions.

steadied at about 55 miles per hr from the west northwest. The exact time of this occurrence is not known, but the total change from an 85-mile southeast wind to a 55-mile west northwest wind probably occurred in less than 3 min. Similar rapid changes occurred on several other occasions and probably account for some of the marked variations in direction noted in Table 1.

Table 9 gives the average velocity of all winds of 30 miles per hr or more, grouped by direction. It shows fairly consistent ratios between the observed velocity at the Empire State Building and the average velocity for a 1-hr period at the other three observatories where recording instruments were used. The velocities shown in the right-hand column are generally about 60% to 65% as great as observed velocities at the Empire State Building, except in certain cases in which the skyscrapers of lower Manhattan shield the anemometer of the U. S. Weather Bureau. Thus, with winds from the northeast the ratio is 33%, and with winds from the east northeast, the ratio is only 21 per cent.

Gust Velocities.—The observations made during this program show clearly that the building not only deflects in accordance with the average wind velocity,

but also vibrates due to variations in velocity. During a strong wind the anemometer in the Empire State Building moves back and forth continuously over a range of about 10 miles per hr, but occasionally a vigorous puff or a sudden lull may cause a momentary change in velocity as much as 20 miles per hr above or below the average velocity.

The velocity and duration of these momentary gusts are of considerable importance, as a 40% increase in velocity would cause an increase in pressure of nearly 100%, and if this sudden increase should occur in harmony with the vibration of the structure, the resulting stresses would be much greater than with a steady wind. On the other hand, however, it is quite probable that each gust of wind occurs over a relatively small area and therefore would not change the average pressure as much as indicated by the velocity-pressure equation. To illustrate this point the writer remembers one occasion when an observation was being made on the 5 manometers on the south side of the fifty-fifth floor. Readings had just been recorded for Stations 1, 2, 3, and 4 (Fig. 4) and all were less than 0.50 in. At Station 5 there was a difference of nearly −0.50 in., but while it was being observed, the difference suddenly increased to about −1.50 in. and remained steady at this point for about 10 sec. During this time several quick glances at the other manometers showed no great change at any of the other stations. This would seem to indicate that a gust velocity might be two or three times as great as the average velocity but that the gust might last only a brief time and extend over a relatively small area.

The data in Table 1 indicate that at the U. S. Weather Bureau in New York, N. Y., the maximum velocity for 5-min periods is generally about 17% greater than the 1-hr average. During the New England hurricane of September 21, 1938, the Harvard Meteorological Observatory records indicated an average velocity of 83 miles per hr for 1 hr; 94 miles per hr for $\frac{1}{2}$ hr; 111 miles per hr for a 5-min period; and gust velocities of 173 and 186 miles per hr. The velocity for the maximum 5-min period, therefore, was 34% greater than for the maximum 1-hr period, and the maximum gust velocities were more than twice the maximum hourly velocity.

Wind Pressure.—It is recommended that the data on actual wind pressures be studied in conjunction with the report of the wind-tunnel tests by Messrs. Dryden and Hill.[3] Unfortunately these tests did not include models of the surrounding buildings; but in spite of this the writer feels that their data are more complete and more reliable than the data actually obtained at the Empire State Building.

The Dryden-Hill tests indicated: (*a*) That the greatest overturning moment occurs when the wind blows directly against one face; (*b*) that a suitable value for use in design of tall buildings is 0.0038 V^2, in lb per sq ft (V = wind speed, in miles per hour); and (*c*) that the velocity 15 ft above the top of the observation tower is approximately 23% greater than the velocity of the approaching wind. From the model tests, it was also possible to measure, accurately, the decrease in the overturning moment as the wind becomes more diagonal. The relations between the direction of the wind and the overturning wind force are

[3] "Wind Pressure on a Model of the Empire State Building," by Hugh L. Dryden and G. H. Hill, *Research Paper No. 525,* National Bureau of Standards.

shown in Fig. 25 and have been used in analyzing the plumb-bob and collimator readings.

The Fire Tower.—The vent shaft built in conjunction with the fire tower acts like an enormous chimney and causes a strong up-draft at all times. This

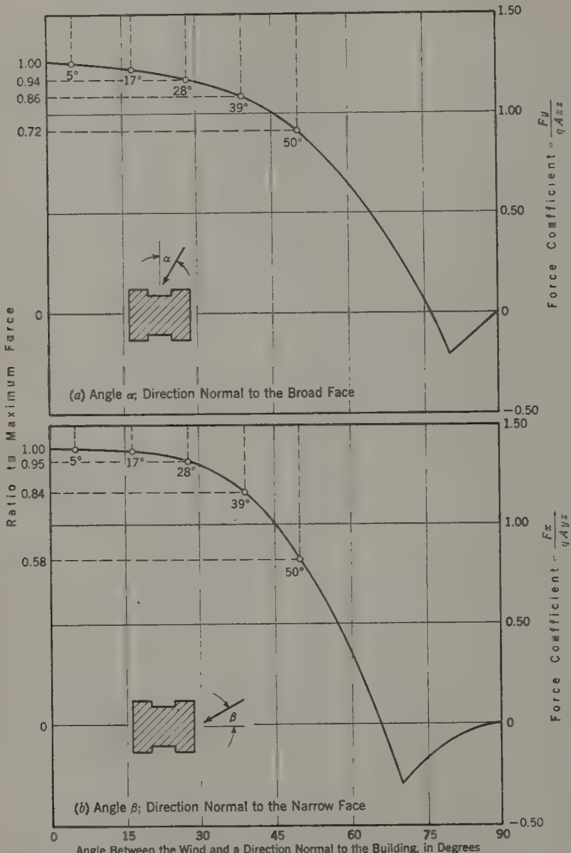

FIG. 25.—RELATION BETWEEN DIRECTION OF WIND AND OVERTURNING FORCE

shaft extends from the sixth floor to the roof and has an unobstructed opening of about 40 sq ft at the top. When the building was first completed, the up-draft in the vent shaft was so strong that leakage under the exit doors caused an objectionable high-pitched whining noise. In order to reduce this noise all doors leading to the fire tower were weather-stripped. This reduced the

noise but did not eliminate it, and in March, 1933, a light bulkhead was built entirely across the vent shaft at the fifty-fourth floor.

This bulkhead reduced the noise to a satisfactory extent, but when a door was opened anywhere in the fire tower, the sound of the wind in the vent shaft was reduced immediately. After becoming aware of this phenomenon, the writer was able to tell by sound if all doors to the fire tower were tightly closed, and the plumb-bob observations were not made unless all doors were shut, as an open door would seriously affect the plumb-bob. The movements of the plumb-bob, shown in Fig. 12, may have been caused by temporary drafts, due to occasional opening of doors to the fire tower, rather than by deflection of the building.

There is also an up-draft on the exterior on the windward side near the top of the building, and on sunny days the heat of the sun warms one or two faces and the warm air rises with a perceptible velocity. On several occasions the writer has walked around on the seventy-fifth floor, successively opening each window a few inches to note whether the wind was blowing in or out. These observations generally showed a strong outward draft at about 75% of the windows. On one occasion when this test was made, the outward draft was noticed at all except seven windows, indicating a negative pressure on about 91% of the periphery at the seventy-fifth floor.

The Plumb-Bob.—At first the plumb-bob had a tendency to wander in an irregular manner due to drafts within the vent shaft. When all the doors leading to the fire tower were closed, the strong up-draft in the vent shaft produced a peculiar whistling noise; but if any of these doors were opened, this noise decreased greatly and the gyrations of the plumb-bob increased, due to the disturbing effect of the draft from the open door. These difficulties were solved by attaching a pair of oil dampers to the plumb-bob as shown in Fig. 11, and by recording only those observations that were made when all doors to the fire tower were closed.

In making plumb-bob readings the dampers were immersed in oil until the plumb-bob was moving in an ellipse with the major axis about 2 in. long. Small markers were placed on the target to mark the ends of the major and minor axes of this ellipse. As soon as the plumb-bob was observed to make three or four complete circuits around the same ellipse, the center was observed and recorded. This process was repeated until at least ten readings had been made. These readings were then averaged and entered in the records as a single point for that particular wind.

Deflection of Building from Plumb-Bob Readings.—The deflection of the building was studied by plotting the plumb-bob observations on eight charts, such as the two in Fig. 26. Since the wind force is directly proportional to the square of the velocity, the indicated velocity of the wind was plotted on a parabolic scale so that the deflection curve on these charts will be a straight line. Winds of less than 30 miles per hr were omitted, as the force of these winds apparently was not sufficient to overcome the internal friction in the masonry materials.

A study of these charts showed that the center of the building is at 6.40 on the east-west axis. The center on the north-south axis is not so well

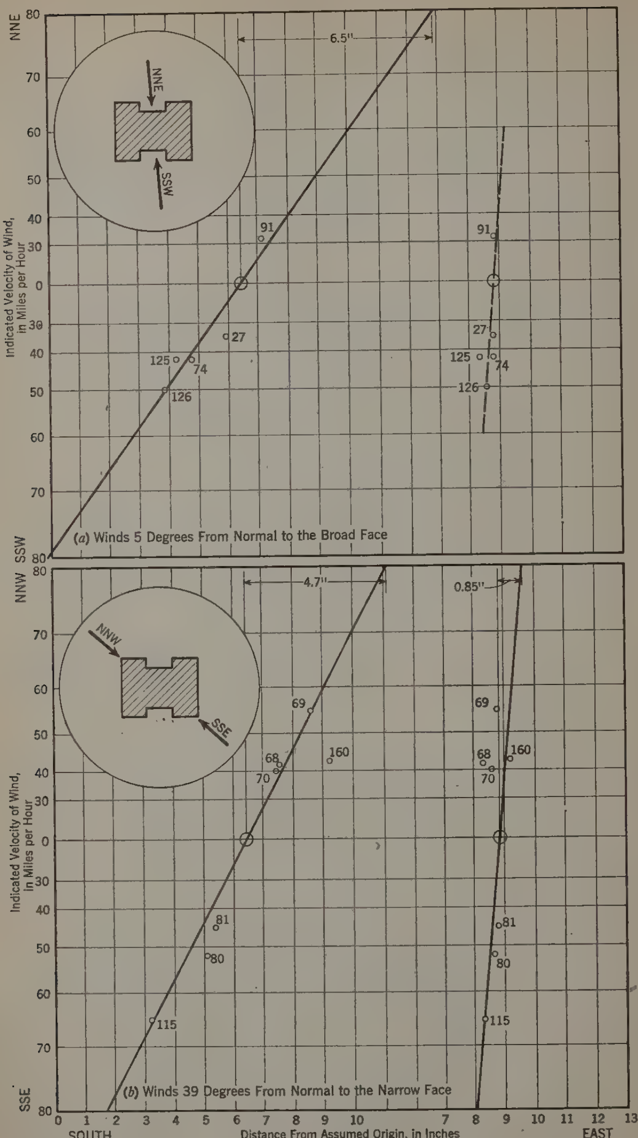

FIG. 26.—DEFLECTION OF THE BUILDING, INDICATED BY PLUMB-BOB

defined, however, due to the fact that the building is much stiffer in this direction, and the plastered partitions and other masonry probably were not seriously cracked in this direction until Storm 109 (see Table 1) on November 9, 1932. The few observations made after this date indicate that the east-west center is

TABLE 10.—DEFLECTION OF BUILDING, FROM PLUMB-BOB OBSERVATIONS

Item No. (see Table 1)	Velocity of wind, in miles per hour	Direction of wind at Empire State Building	Angle between wind and normal to building	Force ratio*	Observed deflection, in inches	"Normal" deflection, in inches	Item No. (see Table 1)	Velocity of wind, in miles per hour	Direction of wind at Empire State Building	Angle between wind and normal to building	Force ratio*	Observed deflection, in inches	"Normal" deflection, in inches
(a) WINDS AGAINST NORTH FACE; ORIGIN ESTIMATED AT 6.40							(c) WINDS AGAINST EAST FACE; ORIGIN ESTIMATED AT 8.85						
31	35	N	28	0.94	1.00	1.06	80	52	SSE	39	0.84	0.15	0.18
68	42	NNW	50	0.72	1.12	1.56	100	75	SE	17	0.99	0.65	0.66
69	55	NNW	50	0.72	2.24	3.12	101	75	SE	17	0.99	0.97	0.98
70	40	NNW	50	0.72	1.08	1.50	109	60	ENE	50	0.58	0.65	1.12
89	35	NE	17	0.98	1.11	1.13	110	60	ENE	50	0.58	0.56	0.96
90	42	NE	17	0.98	1.11	1.13	115	65	SSE	39	0.84	0.49	0.58
91	32	NNE	5	1.00	0.73	0.73	129	58	ENE	50	0.58	0.13	0.22
109	60	ENE	39	0.86	3.30	3.84	151	35	SE	17	0.99	0.45	0.45
110	60	ENE	39	0.86	3.36	3.91	153	57	SE	17	0.99	1.65	1.67
129	58	ENE	39	0.86	2.81	3.26	154	55	E	28	0.95	1.60	1.68
(b) WINDS AGAINST SOUTH FACE; ORIGIN ESTIMATED AT 6.40							(d) WINDS AGAINST WEST FACE; ORIGIN ESTIMATED AT 8.85						
27	35	SSW	5	1.00	0.50	0.50	69	55	NNW	39	0.84	0.05	0.06
61	32	S	28	0.94	0.65	0.69	94	50	WNW	5	1.00	0.70	0.70
62	38	S	28	0.94	0.80	0.85	103	42	WNW	5	1.00	−0.17	−0.17
74	42	SSW	5	1.00	1.62	1.62	104	45	WNW	5	1.00	−0.40	−0.40
77	42	S	28	0.94	1.95	2.07	105	50	WNW	5	1.00	−0.32	−0.32
80	52	SSE	50	0.72	1.35	1.87	124	50	WSW	50	0.58	0.54	0.93
81	45	SSE	50	0.72	1.00	1.39	130	47	NW	17	0.99	0.04	0.04
112	35	SW	17	0.98	1.20	0.22	131	40	WSW	50	0.58	0.34	0.59
115	65	SSE	50	0.72	3.14	4.36	132	35	W	28	0.95	0.40	0.42
124	50	WSW	39	0.86	2.17	2.53	133	40	NW	17	0.99	0.25	0.25
125	42	SSW	5	1.00	2.17	2.17	134	46	NW	17	0.99	0.03	0.03
126	50	SSW	5	1.00	2.58	2.58	155	90	NNW	39	0.84	1.85	2.20
131	40	WSW	39	0.86	1.01	1.17	156	55	NW	17	0.99	0.95	0.96
....	....	....	....	....	....	....	157	55	NW	17	0.99	0.95	0.96
....	....	....	....	....	....	....	160	42	NNW	39	0.84	0.40	0.48
....	....	....	....	....	....	....	161	45	NW	17	0.99	0.45	0.45

* From Fig. 25.

probably between 8.70 and 9.00 on the plumb-bob target and an average value of 8.85 has been assumed.

The wind-tunnel tests previously mentioned[3] showed that the greatest overturning moment occurs when the wind is blowing directly against one face of the building, and that this overturning moment gradually decreases as the wind becomes more diagonal (see Fig. 25). The deflections caused by diagonal winds, therefore, can be increased to show the deflection that would have occurred if the wind had been directly against one face of the building. Thus, when the wind is 28° from a normal to the broad face, the overturning force is about 0.94 times as great as for a normal wind and the observed deflection

should be divided by 0.94 to obtain the maximum deflection for that particular velocity.

Table 10 was obtained in this manner. This study shows that an indicated velocity of 80 miles per hr directly against either of the broad faces causes a deflection of about 6.5 in. The same wind against either of the narrow faces would cause a deflection of about 2.0 in., which indicates that the moment of inertia about the north-south axis is more than three times as great as about the east-west axis. This ratio may not be a constant, however, as the plastered partitions and other masonry probably carry a large proportion of the wind load when the wind is against the narrow face, and each severe storm may cause further cracking and more flexibility in this direction.

If the indicated velocity at the top of the observation tower is assumed to be about 23% greater than the velocity of the approaching wind, the average deflection of the building at the eighty-sixth floor, as indicated by the plumb-bob, is as follows:

Description	Indicated velocity at the top of the building, in miles per hour:	
	80	100
True velocity of approaching wind, in miles per hour	65	81
Deflection of the building, in inches:		
North-south	6.5	10.2
East-west	2.0	3.1

Deflection of Building from Collimator Readings.—A study of the collimator readings given in Table 5 shows that the center is approximately at 5.6 along the north-south axis. Using this center, the collimator readings have been rearranged in Table 11 to show the average deflection, and the maximum amplitude of vibration for winds greater than 30 miles per hr against either of the broad faces. By using the "force ratios" obtained from Fig. 25(a), it is possible to estimate the probable deflection and vibration that would have occurred with a "normal" wind blowing directly against either of the broad faces.

Columns (7) and (8), Table 11, when plotted, indicate that the north-south deflection of the building is about 6.5 in. with an indicated wind velocity of 80 miles per hr. The average deflection indicated by the plumb-bob and the collimator is in close agreement for north-south deflections. In the east-west direction, however, there are insufficient collimator readings to determine the deflection with any degree of assurance.

In addition to the average deflection, the building vibrates, and the maximum range of vibration, as indicated by the collimator readings, is about 10% greater than the average deflection. This means that an 80-mile wind may cause the building to deflect about 6.5 in. and to vibrate about 7.2 in. about the deflected position. In this case the extreme deflection of the building would be $6.5 + 3.6 = 10.1$ in.

Fig. 14 shows graphically the period of vibration of the building as observed with the collimator on November 1, 1932; that is, there were 104 cycles in about 860 sec, which indicates a period of vibration of about 8.28 sec instead of 8.38

sec as reported by Professor Rathbun. It is also difficult to understand how a period of 8.14 sec was derived from the data in Fig. 15.

In December, 1938, when the writer last visited the building, the wind velocity was between 40 and 50 miles per hr, and it was possible to hear faint

TABLE 11.—DEFLECTION OF BUILDING FROM COLLIMATOR READINGS

Item No. (see Table 5)	Velocity of wind, in miles per hour (1)	Direction of wind at Empire State Building (2)	Angle between wind and normal to building (3)	ACTUAL OBSERVATIONS WITH COLLIMATOR: Average deflection, in inches (4)	ACTUAL OBSERVATIONS WITH COLLIMATOR: Maximum vibration, in inches (5)	Force ratio* (6)	"Normal" deflection, in inches (7)	"Normal" vibration, in inches (8)
(a) WINDS AGAINST NORTH FACE; ORIGIN ESTIMATED AT 5.6								
107	52	ENE	39°	1.10	3.0	0.86	1.3	3.5
108	60	ENE	39°	2.75	4.7	0.86	3.2	5.5
129	58	ENE	39°	3.45	4.3	0.86	4.0	5.0
(b) WINDS AGAINST SOUTH FACE; ORIGIN ESTIMATED AT 5.6								
117	35	SW	17°	2.6	0.6	0.98	2.7	0.6
119	34	SSW	5°	1.5	0.8	1.00	1.5	0.8
126	51	SSW	5°	2.6	2.0	1.00	2.6	2.0

* From Fig. 25.

groans as the building vibrated; 28 groans were heard in slightly less than 2 min, timed with an ordinary watch, which seems to indicate that the period of vibration is still about 8.3 sec as measured on November 1, 1932.

Conclusions.—From the foregoing, the writer advances the following nine conclusions:

(1) The plastered partitions, concrete floors, and masonry walls add both strength and stiffness to the building and carry a large percentage of the wind load. Any theory that ignores this fact is probably so far from the truth that great refinement is not warranted.

(2) The ratio between the stiffness of the building and the stiffness of the steel frame is not constant. It may have been 10 to 1 when the building was first completed and it may still be about 3 to 1. It is quite probable, however, that extremely severe storms will extend the existing cracks and allow the steel frame to carry a greater proportion of the wind load.

(3) After a severe storm, the building does not return exactly to normal. As the wind decreases, the deflection decreases until the frictional resistance of the building equals the restoring force of the deflected materials. The building then remains in this slightly deflected position until the next storm.

(4) The plumb-bob readings show that an indicated wind velocity of 80 miles per hr against either of the broad faces caused a deflection of about 6.5 in.; but the same velocity against either of the narrow faces caused a deflection only one-third as great.

(5) The collimator readings show that the building vibrates about the deflected position. The period of vibration is about 8.3 sec, and the range of

vibration is slightly greater than the average deflection. An indicated velocity of 80 miles per hr, against either of the broad faces, therefore, would cause the building to vibrate back and forth over a range of about 7.2 in., and the extreme deflection would be $6.5 + 3.6 = 10.1$ in.

(6) During severe storms the vibration of the building causes groans as the masonry slips on the steel frame, or along cracks caused by previous storms.

(7) There is generally a strong up-draft on the windward side of the Empire State Building above the fifty-fifth floor. This up-draft, combined with the horizontal wind, probably causes a resultant velocity considerably greater than the velocity of the approaching wind. Indicated wind velocities at the Empire State Building, therefore, should be reduced to obtain the true wind speed.

(8) Both the velocity and direction of the wind are seriously affected by surrounding buildings. Low buildings receive considerable protection from surrounding buildings, and the sheltered area on the lee of a building is relatively greater during more severe storms.

(9) In New York City, during a very severe storm, there is considerable difference in velocity at points 100 ft, 500 ft, and 1 000 ft above the ground. A rough estimate indicates that simultaneous velocities at these three levels might be 55, 70, and 80 miles per hr. The corresponding wind pressures, therefore, would be about 20.9, 26.6, and 30.4 lb per sq ft, respectively, at elevations 100 ft, 500 ft, and 1 000 ft above the ground.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

MECHANICAL STRUCTURAL ANALYSIS BY THE MOMENT INDICATOR

Discussion

BY OTTO GOTTSCHALK, ESQ.

OTTO GOTTSCHALK,[11] ESQ. (by letter).[11a]—As its name implies, the moment indicator permits the abstract analysis of stresses, but, without proper and direct reference to actual structures. In dealing with abstract stresses, the same as with abstract mathematics, the investigator frequently derives beautiful and harmonious results, which then require mechanical interpretation. The moment indicator is such a device, and the author's optimism is understandable. However, the writer feels a certain skepticism regarding a few aspects of the problem. In 1937,[12] the writer emphasized the inconsistency and confusion inherent in the practice of analyzing structures mathematically, in terms of abstract stresses, as if those stresses, themselves, were part of the structure. Unfortunately, this fundamental fallacy is being extended to mechanical analysis with equally fallacious results.

In mechanical stress analysis, the model generally is loaded and the bending moments are computed from the measured rotation of the tangent to the deflection curve. In mechanical structural analysis (as distinct from stress analysis), the model is subjected to a unit displacement or rotation at the section being studied. The deformation of the model results in a physical graph of the influence line on a scale, such that the ordinates can be read at the places where the loads are acting. The latter is the correct approach and is the basis of the Beggs deformeter gage mentioned by the authors.[4]

The authors have developed valuable formulas for correcting errors in determining tangents. Rigid exactness of individual tangents, however, depends so much upon perfect homogeneity that the slightest microscopic imperfection in the material or the device clamped upon it will produce errors

NOTE.—This paper by Arthur C. Ruge, M. Am. Soc. C. E., and Ernst O. Schmidt, Esq., was published in October, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1939, by Messrs. John B. Wilbur, and William J. Eney.

[11] Buenos Aires, Argentine Republic.

[11a] Received by the Secretary March 13, 1939.

[12] "Structural Analysis Based Upon Principles Pertaining to Unloaded Models," by Otto Gottschalk, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1019.

[4] *Loc. cit.*, Vol. 88 (1925), p. 1208.

which are magnified still more by levers and light rays. The use of metallic devices and a relatively soft model material (the true homogeneity of which can be only surmised) would certainly require a degree of precision control that cannot be maintained outside of permanent and special laboratories.

On the other hand, ordinates can be made relatively large by mechanical devices, and quite in accord with abstract theories. They may be considered as the sums of individual deformation, in which local errors or variations are counterbalanced. Perhaps that is the mechanical explanation for the fact that tangent methods of analysis have failed to compete with mechanical methods involving ordinate reading. Although it certainly seems to be a favorable characteristic of the moment indicator that it does not involve the reading of tangent changes, but merely the relation between such changes, this in itself does not guarantee that errors in observing two tangents, relatively, may not be cumulated. The usefulness of any instrument for mechanical analysis is impaired if the instrument requires microscopic reading. This fact in itself tends to take the instrument out of the hands of the analyst and place it into the care of some subordinate or assistant. In the analysis of a special structure of some magnitude it might be justifiable to spend the time cutting and investigating five or ten separate models in order to compare the consistency of results; but for daily routine, it will generally be considered an unjustified burden on the designing staff. The statement in the paper to the effect that the moment indicator requires less manipulative skill than other available methods of mechanical analysis, capable of giving comparable accuracy, is a sweeping one to which much can be said in refutation.

For the past one hundred years or more, the entire general theory of statics has been submerged in a veil of mystery by a mass of artificial, intricate mathematical functions of stress. The effect has been to obscure the simple geometrical relation that can be observed by eye. Similarly, in mechanical analysis, the present tendency is to solve structural problems by microscopic examinations which can be visualized macroscopically, more easily and more efficiently. To demonstrate his point, the writer must refer to a device first described in 1926.[13] Similar devices for studying the effect of vertical loads,[14] and for wind stresses,[15] have been described by the writer subsequently. These devices have demonstrated how the theory of statics may be simplified fundamentally by a geometrical interpretation of the visible natural behavior of the given structure.[12] A third fundamental application of the splines and metal clamps recommended by the writer is to the analysis of the structural element with which the designer deals every day. The basic principles underlying the writer's views on this subject have been demonstrated a number of times. The fundamental operations for a structural analysis may be demonstrated briefly by reference to Fig. 14, which is a continuous beam of constant cross-section. With the load shown in Fig. 14(a), only a single unit displacement is required to supply the influence line of the reaction R_c in Fig. 14 and the shear effect, F_E, shown in Fig. 14(b). To the left of Point A, in Fig. 14(a)

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[13] *Journal,* Franklin Inst., July, 1926, p. 61, and February, 1929, p. 245.

[14] *Transactions,* Am. Soc. C. E., Vol. 103 (1938), Fig. 30, p. 1067.

[15] *Proceedings,* Am. Soc. C. E., December, 1938, Fig. 5, p. 2030.

a load P_1 is acting on a cantilever for which the influence line is the tangent of the curve at Point A. When Beams $A\ B\ C\ D$ in Fig. 14(*a*) are fixed at Point A, the influence line is found by simply lifting the extended end beyond Point A, until the tangent at Point A is horizontal, as shown by the dotted line in Fig. 14(*a*). Fig. 14(*d*) demonstrates how the influence line for the bending M_F, at Point F, is obtained by joining two similar splines by a standard moment clamp and running these lines through the points of support until the

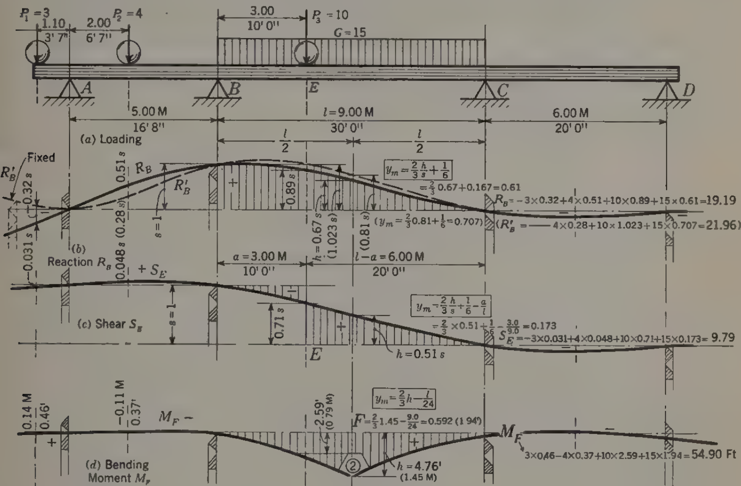

Fig. 14.—Structural Analysis and Stress Calculation of a Continuous Beam

apex of one clamp rests below Point F. Without using any special degree of precision, and independent of local conditions, the observed result of this analysis is well within 3% of those found by theoretical computations. The same degree of accuracy will be obtained for reactions and shears (Figs. 14(*b*) and 14(*c*)) by applying two transverse displacements equal to one-half unit, without moving the lines horizontally; that is, the support at Point A, or at Points A and B, respectively, is first lifted and then lowered through one-half unit displacement. It should be evident that results obtained in this manner will require less time than would be necessary even to adjust microscopes. When the section of a beam varies in adjacent spans, splines may be added or the spans of the model may be reduced accordingly.

The system is applicable equally to complex frames in steel or reinforced concrete. It can be applied to typical tunnel cross-sections, and to other common elements such as are encountered daily in the modern structural engineering office. No promoters of mechanical equipment for structural analysis should lay claim to merit unless their device can be made to show this range of practical usefulness.

In general, the authors are to be commended for their thorough investigation of model material, and for the formulas introduced to determine necessary corrections. Furthermore, by fastening the moment indicator to the members of the model instead of incorporating it as a part of the member, the authors have, in principle, solved the complex problems of handling. The writer feels, however, that the authors have not maintained a sufficiently clear distinction between the fundamental concept of a mechanical structural analysis, and a mechanical stress analysis as defined roughly in this discussion. The authors have confined themselves to the latter, whereas the writer is an advocate of the former.

Correction for *Transactions:* In Fig. 13(*e*), at End A, change "6.647" to "6.447"; and, "0.260 in." to "0.460 in."

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

SIPHONS AS WATER-LEVEL REGULATORS

Discussion

By Henry R. King, Esq.

Henry R. King,[17] Esq. (by letter).[17a]—In 1933, Mr. Stevens called attention to the fact that "once it has started, the flow through any siphon may * * * diminish to a small part of its normal capacity without breaking of its siphonic action." Other writers have also discussed this phenomenon.

In its Southwest Sewage Treatment Works, the Sanitary District of Chicago, Ill., has three large siphon spillways which serve as excess flow and emergency by-passes. The spillways are designed to prime with water surfaces within 18 in. of the tops of channels and open tanks and to operate at any rate from zero to full discharge. Therefore, they may be classified as water-level regulators. The design was based on data obtained from small model tests made in 1935 to determine whether a siphon was suitable for this kind of service. The type of spillway tested (see Table 6) was found to be quite satisfactory. The overflow crest functioned as a weir until the flow was sufficient to raise the water surface to the bottom edge of the primer plate, at which elevation the level remained constant, the siphon inhaling air for the larger flows up to full discharge.

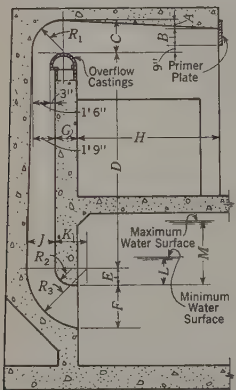
Table 6 gives the capacities, number of siphons, and dimensions for the large spillways at the Southwest Sewage Treatment Works. Each consists of a number of identical siphons in which all primer plates are set at the same elevation. The overflow crest is made of castings that can be raised for the installation of other castings under them—an essential provision in order to keep water levels down until after the addition of possible future stages in the sewage treatment process. The accessibility of the overflow crests makes this elevation adjustment possible. The overhang of the overflow castings is intended to serve as a nappe aerator.

Note.—This paper by J. C. Stevens, M. Am. Soc. C. E., was published in October, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1939, by Messrs. A. Griffin, and H. P. Currin and D. M. Umphrey; and April, 1939, by T. J. Corwin, Jr., M. Am. Soc. C. E.

[17] Senior Civ. Engr., The Sanitary District of Chicago, Chicago, Ill.

[17a] Received by the Secretary April 29, 1939.

TABLE. 6—SCHEDULE OF DIMENSIONS, SIPHON SPILLWAYS, SOUTHWEST SEWAGE TREATMENT WORKS, SANITARY DISTRICT OF CHICAGO, ILL.



Typical Section	Dimension	Siphon Spillway: No. 1	No. 2	No. 3
Capacity (highest discharge water elevation), in cubic feet per second		1 000	2 200	2 500
Number of siphons		5	7	8
(see section above)	Width	5 ft 6 in.	8 ft 0 in.	7 ft 10½ in.
	A	6 in.	6 in.	12½ in.
	B	1 ft 3 in.	1 ft 6 in.	11½ in.
	C	2 ft 6 in.	2 ft 9 in.	2 ft 9 in.
	D	14 ft 9 in.	23 ft 0 in.	26 ft 0 in.
	E	12 in.	1 ft 3 in.	1 ft 6 in.
	F	4 ft 10½ in.	3 ft 2¼ in.	3 ft 8½ in.
	G	1 ft 9 in.	1 ft 9 in.	2 ft 0 in.
	H	6 ft 0 in.	13 ft 0 in.	14 ft 9 in.
	J	3 ft 0 in.	2 ft 0 in.	1 ft 9 in.
	K	3 ft 0 in.	2 ft 6 in.	3 ft 9 in.
	L	1 ft 0 in.	1 ft 0 in.	1 ft 0 in.
	M	8 ft 0 in.	8 ft 0 in.	8 ft 0 in.
	R_1	2 ft 6 in.	2 ft 6 in.	2 ft 9½ in.
	R_2	1 ft 3 in.	1 ft 3 in.	1 ft 6 in.
	R_3	6 ft 0 in.	6 ft 0 in.	5 ft 6 in.

The wide variation of water depth in the discharge conduits of these spillways (see Table 6) will sometimes cause considerable depth of seal on the siphon outlets. Therefore, appreciable priming heads are required, and these determine the maximum water levels in the forebays. The lower edges of the primer plates have been set at approximately the priming head elevations. This means that when the water level is just below the primer plate, the free weir discharge over the siphon crest will be sufficient to provide a minimum water velocity, probably about 0.8 ft per sec, which will carry the entrained air down to the siphon outlet. For these spillways the discharge will reach about 5% of capacity before the water level rises to the primer plates, which means a lower relative range of siphonic discharge than was obtained for the Walterville siphons, with a minimum discharge of about 3% of capacity.

If the depth of seal on a siphon outlet is considerable, the ratio of the downward flow area to the width at the crest should not be too great if the priming head is to be limited; but if the depth of seal is only a few inches, as in the case of the Walterville siphons, priming heads are small and the ratio of flow area to crest width is probably not of much importance. The siphonic action, however, broke on the Walterville siphons at a water rate of about 0.9 cu ft per sec per sq ft of downward flow area.

When a siphon is inhaling air, it is functioning as an hydraulic air compressor. For this reason, experiments made by the writer on a small hydraulic

air compressor, using a more elaborate type of water-level regulator, are of considerable interest in this connection.

The experimental set-up is shown in Fig. 10. Air entered the down-draft tube just below the siphon crest from several pipes, with perforated inverts,

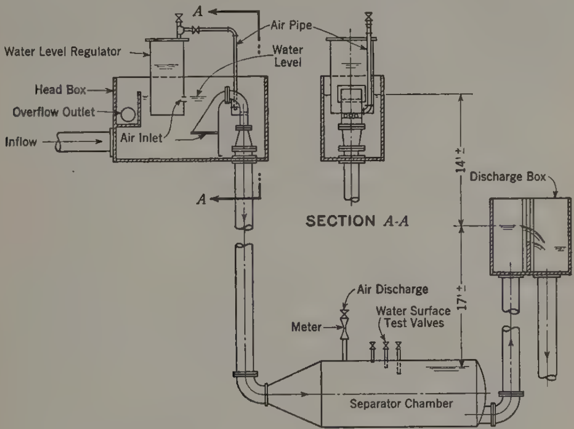


Fig. 10.—Experimental Hydraulic Air Compressor

extending across the flow section. These pipes were supplied from an air chamber, under the crest, which was connected by pipe to the top of the water-level regulator, a chamber with open bottom partly submerged in the water in the headbox. The restricted area between the perforated pipe produced downward nozzle-like discharges into the surface of the water below, in the down-draft tube, entraining air which the water carried down into the separator chamber. When the perforated air pipes were supplied from the atmosphere, the water level in the headbox rose as the flow increased. To maintain a constant level in the headbox it was necessary to have, in the down-draft tube above the water surface, a partial vacuum varying with the flow. The water-level regulator produced this effect. The chamber inhaled air through a horizontal slot on one side. The water surface in the headbox remained level with the top of this air inlet for the full range of discharge used in the tests. The rise of the water inside the level-regulator chamber, due to the negative head transmitted from the down-draft tube of the compressor, could be limited by throttling the valve in the air line.

The tests indicated that, in order to avoid surging, the air inlet should be above the invert of the siphon crest and, also, high enough so that the rate of flow in the down-draft tube would increase to about 1.0 cu ft per sec per sq ft of area before the water level in the headbox rose to the top of the air inlet. Then, for the higher flows the water level remained unchanged. The maintenance of a constant water level at the entrance to an hydraulic air

compressor is important, particularly if there is a varying flow and a relatively small working head.

The author's paper has genuine value. Presentation of the results of such careful and complete tests as those made on the large Walterville siphons is highly desirable. Much experimental work remains to be done before the engineer can compute with certainty the capacity of a siphon or similar conduit which includes bends of varying degree of curvature, closely spaced and, in some cases, with the final bend right at the outlet.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

DISCUSSIONS

ANALYSIS OF RUN-OFF CHARACTERISTICS

Discussion

BY MESSRS. C. S. JARVIS, AND HOWARD M. TURNER

C. S. JARVIS,[26] M. AM. SOC. C. E. (by letter).[26a]—In the midst of the usual output of new plans hopefully aimed at refinements in run-off estimates, it is reassuring to find that the author, along with other proponents of new procedures, recognizes and admits the inherent limitations as to accuracy, and consequently the main dependence on logical reasoning, guided by mature experience, in any practical approach to evaluation of run-off factors and characteristics. Of all the divergent curves in Fig. 4, for instance, it is expected that appropriate choice shall be made after sufficient experience and with the aid of detailed topographic maps of the drainage area. However, the limitless variations in rainfall patterns, widely departing from what may have been adopted as the basis of reckoning, are capable of producing some very erratic results.

The writer has attempted to meet a requirement in the field of quantitative appraisal of run-off characteristics as they might be influenced by land-surface treatments and other soil and water conservation measures. The preliminary results are depicted in Fig. 13 and are held as only tentative, pending the completion of a more detailed analysis involving more rigorous procedure and a more nearly scientific approach. The agreement resulting from the two methods was so marked as to establish the preliminary solution nearly on a par with the final result.

Among the assumptions involved in both approaches to this problem were the following:

(1) Retention of rainfall by surface storage and induced infiltration on a representative portion of the drainage basin area, such as one-fourth or one-half, with no escape to drainage channels by over-land or other surface flow,

NOTE.—This paper by Otto H. Meyer, Assoc. M. Am. Soc. C. E., was published in November, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1939, by Messrs. Victor H. Cochrane, and Bertram S. Barnes; March, 1939, by Messrs. LeRoy K. Sherman, Richmond T. Zoch, and Merrill Bernard; April, 1939, by Messrs. Franklin F. Snyder, and W. G. Hoyt; and May, 1939, by C. O. Clark, Jun. Am. Soc. C. E.

[26] Hydr. Engr., SCS, Dept. of Agriculture, Washington, D. C.

[26a] Received by the Secretary April 26, 1939.

must inevitably reduce the total volume of run-off within that flood-rise in about the same proportion;

(2) Reduction of run-off from similar areas by reason of detention works, increased vegetative cover, lengthened course of travel, reduced gradients as well as velocities of flow, and prolonged contact of such water with the soil surface, account for a proportionate decrease of run-off volume; and,

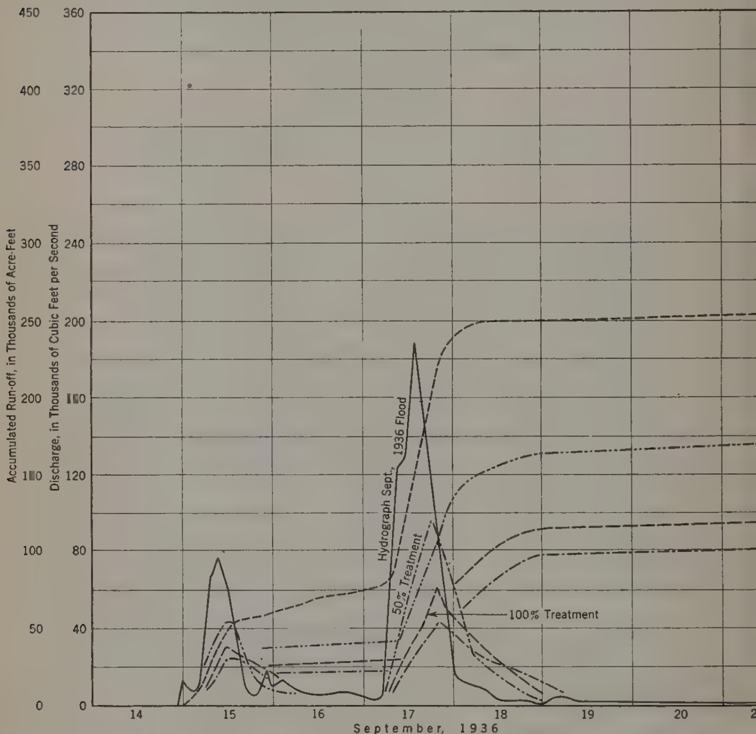
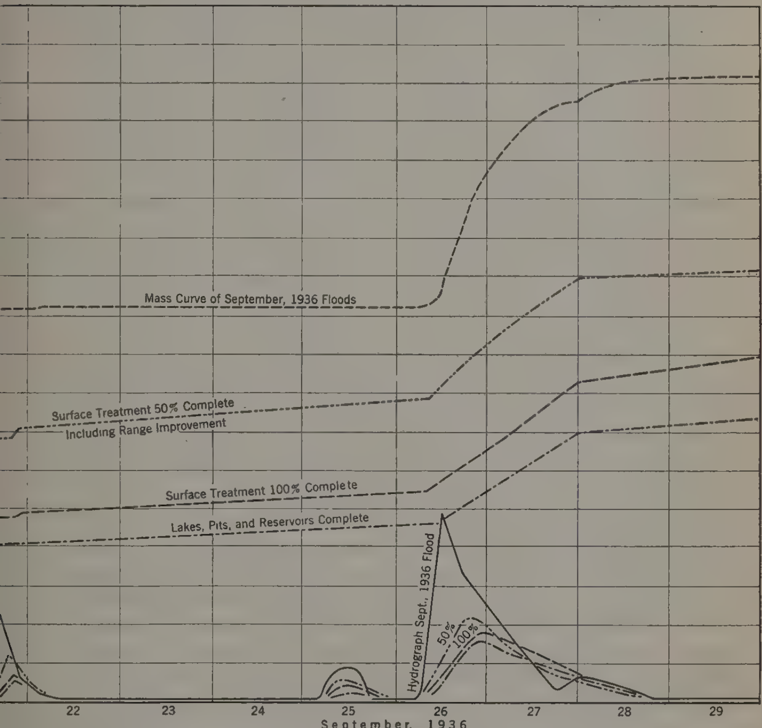


FIG. 13.—HYDROGRAPH AND MASS CURVE OF SEPTEMBER 1936 FLOOD ON NORTH CONCHO
SIVE EFFECTS OF CONSERVATION MEA-

(3) If the treated areas are widely distributed throughout the drainage basin, the base of the flood hydrograph could not conceivably be shortened as a result of such detention and retardation through conservation measures. Due to the reduction in velocities of the component parts of the gathering flood, not only must the occurrence of the flood peak be delayed, but the base of the flood hydrograph must be lengthened. Then the peak discharge (shown graphically in Fig. 13 as the altitude of the triangular form of flood-rise) must necessarily be reduced in measurable amounts both by reason of such lengthen-

ing of the base and by reason of reduced flood volume, represented by area under the hydrograph. Thus, if the area of a triangle is reduced by half and the base remains unchanged, only one-half of the original altitude remains; but if the base is lengthened, the apex is lowered still farther.

Although some percentage of the retained rainfall may appear in the stream channel at some later time, to increase the low-water discharge, the contribu-



RIVER AT SAN ANGELO, TEX.; ALSO, REVISED CURVES TO ILLUSTRATE ESTIMATED PROGRESSURES APPLIED TO THE DRAINAGE BASIN

tions will probably be in the nature of springs or seeps, and generally will be of value in a water-utilization program.

It happens frequently that the apparent lack of hydrologic data may be mended in part by recourse to such procedures and methods of approach as are illustrated in the foregoing example. Briefly, the justification for subtracting a treated area from the rôle of storm run-off contributor depended on the following provision: That the sum of infiltration and available surface-storage depths should equal or exceed the depth of rainfall on that area for any given

period. In this case, soil infiltration records covered a period of several years, and actual performance of level terraces, contour ridges and furrows, diversion and spreading works, and other aids to increased infiltration, under the record-breaking storm conditions of September, 1936, proved their effectiveness in regulating and reducing flood run-off.

HOWARD M. TURNER,[27] M. AM. SOC. C. E. (by letter).[27a]—A "basic hydrograph" is developed, in this interesting paper, which the author defines as the hydrograph resulting from a uniform rain with a duration equal to the time of concentration. A hydrograph for a storm of this length should be a characteristic one for each point. The use of a time unit of subdivisions of the time of concentration instead of days or hours is an interesting suggestion, and the hydrograph itself should prove a useful tool in hydrograph analysis. The author develops this hydrograph as consisting of only two parts, its rising limb (the concentration curve) and its falling limb (the storage curve). He states that there is a sharp break at the top which he has actually found on continuous gage records. Groups of hydrographs of the Pemigewasset River at Plymouth, N. H., reduced to comparative scale, are assembled in Fig. 1. It would be interesting to have the complete information regarding this particular hydrograph and the rainfall producing it.

The writer does not believe that the basic hydrograph given is one which will be produced on a river by a uniform rain storm just equaling the time of concentration, particularly in respect to the sharp break at the top where, in the author's description, the concentration curve changes immediately to the storage curve. This assumes that the stopping of the rainfall all over the water-shed is felt to its full extent immediately at the point of measurement. The writer cannot conceive that the shutting off of the water supply from the rainfall at distant points can possibly be felt so abruptly at the point of measurement, due to the effect of time lag and channel pondage in the entire stream system. The latter is such a powerful factor that it modifies changes in river flow, definitely, through lengths of ordinary channel not involving large storage. The writer has in mind several cases in which continuous stream-flow gagings are made at points a few miles below water-power plants. The sharp, vertical changes in flow at the plant due to closing the station become modified into curved hydrographs with rounded corners and sloping sides a short distance down stream, due to effect of channel pondage.

That this sharp break cannot occur on the "basic hydrograph," except perhaps on very small drainage areas or in very special cases, can be shown simply by dividing any area into its tributary parts. The writer has assumed for this purpose a drainage area as shown in Fig. 14. Any division of the drainage area into tributary areas would serve as well, the selection being purely arbitrary. These tributaries, being smaller than the main river at the point of measurement, must have shorter periods of concentration so that a storm lasting for the period of concentration for the entire area must extend beyond the periods of concentration of the smaller component areas. A storm

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[27] Cons. Engr., Boston, Mass.

[27a] Received by the Secretary May 5, 1939.

beyond the concentration period will consist of a rising concentration curve up to the time of concentration. If the rain continues uniformly beyond this point, the hydrograph will have a horizontal top equal to the maximum reached at the time of concentration, and if the author's assumption is correct, when the rain stops the storage curve will begin with the sharp break he has described. In other words, for a storm longer than the concentration period, the hydrograph, according to the author's concept, must be the same as his basic hydrograph except that it will have a flat place at the top equal in length to the time that the rainfall exceeds the time of concentration.

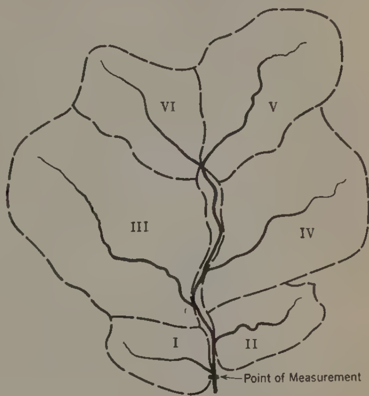

FIG. 14.—ASSUMED DRAINAGE AREA

On this basis the writer gives in Fig. 15 such flood hydrographs adapted from the author's basic hydrograph for the various subdivisions of the drainage area considered. These hydrographs of the tributaries are all drawn with similar "concentration curves" and corresponding "storage curves." They need not be uniform, of course, and in an actual case would not be so; but, on the author's assumption, they would present the general form given with the sharp break at the end of the rain.

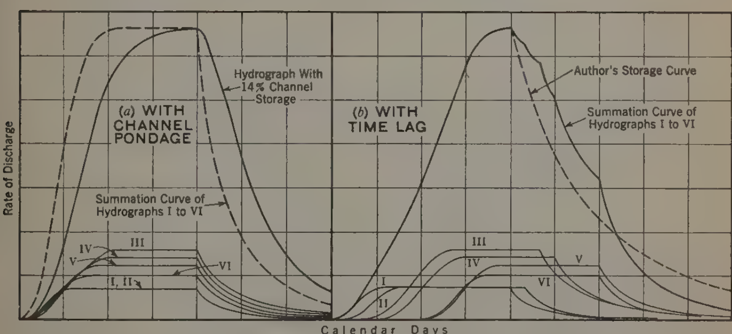

FIG. 15.—FLOOD HYDROGRAPH FOR AREA SHOWN IN FIGURE 14

The flow at the measuring point must be some form of summation of the flows from these separate drainage areas. The writer has combined them in two ways—one as shown in Fig. 15(*a*), allowing time lags for the water from each tributary to reach the point of measurement, and Fig. 15(*b*), allowing no time lag but assuming the main river, from the point of entrance of the upper-

most tributaries to the point of measurement, is a pondage basin. In the latter case, water entering at any point of the basin would be felt immediately at the point of measurement—that is, the outlet of the basin. What actually happens is supposedly a combination of the two, with the storage-basin effect giving a decreasing time of transit along the main river as the river becomes more full. The two assumptions cover the extremes.

Considering Fig. 15(*a*), the descending limb of the curve is of a very different shape from the curve presented by the author. Because of the time lag required for the discharge from each tributary to reach the point of measurement, the storage curve does not begin sharply at the end of the storm. If the sharp curve does not exist at the point of measurement, neither does it exist on any of the tributaries, each of which may be similarly subdivided. If the tributary hydrographs do not have these sharp corners, the cusps in the combined hydrograph will disappear, and the curve will present a smooth descending limb, but with no sharp break.

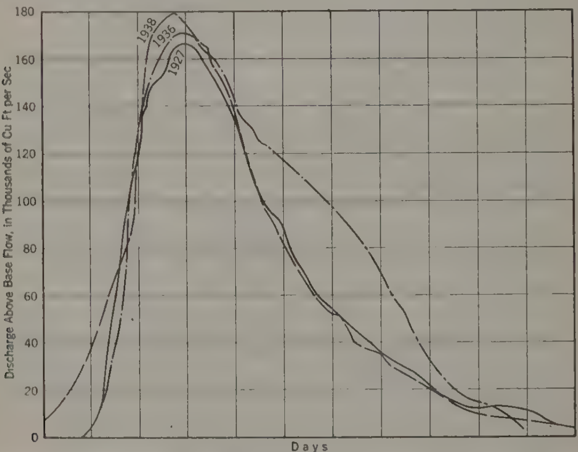

FIG. 16. FLOOD HYDROGRAPHS ABOVE BASE FLOW—CONNECTICUT RIVER AT SUNDERLAND, 1927, AND MONTAGUE CITY, 1936 AND 1938

In the case shown in Fig. 15(*b*), the same hydrographs of the tributary drainage areas have been added without any time lag but with allowance for channel pondage in the main stream from the entrance of the uppermost tributary to the point of measurement, in this case assumed at the peak to be 14% of the total run-off. The effect of this storage is to give a convex curve, instead of the sharp break shown by the author, at the upper part of the hydrograph after the concentration period when the rain is assumed to stop. As in the case in the hydrograph shown in Fig. 15(*a*), if the tributary hydrographs do not have the sharp breaks shown by the author (which, by the same reasoning, they do not have) the descending limb in this case will depart still further from the author's.

It is not maintained that either the results shown in Fig. 15(*a*) or Fig. 15(*b*) represent the actual hydrograph for the assumed case, but neither of these hydrographs of the limiting cases shows the sharp break.

In the writer's opinion the flood hydrograph of a river is much less sensitive than many of the theories of hydrograph synthesis assume, due mostly to the effect of pondage in the entire system. As an example, Fig. 16 shows the flood hydrographs of the three large floods on the Connecticut River at Sunderland, Mass., in 1927 (8 000 sq miles of drainage area) and Montague City, Mass., in 1936 and 1938 (7 890 sq miles). The base flow has been deducted from these in each case by a straight line connecting the flow at the beginning and end of the flood period; but no other corrections have been made. The flood of 1927 was caused by a heavy two-day rainfall concentrated in Vermont, with the center of the storm 60 miles up stream from the point of measurement. The 1936 flood flow was due to a storm lasting four days, quite generally spread over the entire area, combined with melting snow. (The bulge on the recession side was due to additional rainfall occurring after the flood peak.) The 1938 flood was caused by five days of rainfall, of which the last two were extremely heavy in the lower part of the drainage area. The center was in Massachusetts immediately down stream from the measuring point. It may be that it is a matter of chance that the distribution and times of these rainfalls were such as to produce these similar hydrographs. However, the writer is inclined to the theory that the storage on a basin integrates storms of different durations and concentrations and tends to produce quite similar hydrographs for similar flow volumes.

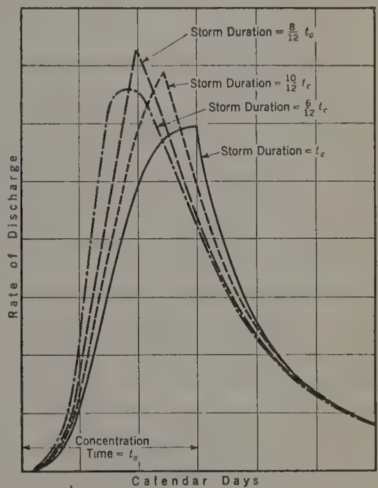

FIG. 17.—"BASIC HYDROGRAPH" AND CORRESPONDING HYDROGRAPHS FOR STORMS OF SHORTER DURATION BUT WITH SAME TOTAL RAINFALL

This is quite at variance with the author's basic hydrograph theory for a similar run-off taking place in 6, 8, 10, or 12 twelfths of the concentration time. Fig. 17 shows the author's hydrographs of floods of varying times taken from his Fig. 2 and multiplied by the proper factor so that each will represent the hydrograph of a flood from storms of the same magnitude but occurring at different times. On this basis the flood produced by a storm of the same size, but in 83% of the time of concentration, will produce a very different shape of hydrograph, with a peak 15% greater. The same storm occurring in 67% of the time will produce a still different hydrograph with a peak 20% higher. This appears to the writer to be a much finer distinction than is found in actual

flood hydrographs. He believes that the storage for the entire length of the main river and of its tributaries through the entire system, and the time of travel of the flood into, through, and out from such storage, tend to cause hydrographs of similar shape for flood volumes of the same size, although there may be quite large differences in the length and distribution of the storm rainfall on the drainage area.

The author is in error regarding the history of modern flood hydrograph analysis. It was the report of the Committee on Floods of the Boston Society of Civil Engineers[28] at first revealed the fact that hydrographs from isolated storms had practically the same length of base. It contains the following statement:[28] "the Committee shows that a flood hydrograph once determined for a given river, even for an ordinary flood, will serve as a basis for the estimation of greater flood run-off, due to the fact that the base of the flood hydrograph (or time of flood period) appears to be approximately constant for different floods." On the next page exceptions are pointed out, among them, "the effect of storms which are of longer duration than the concentration period."

If individual recognition is to be given for this it should go mostly to Mr. S. S. Kent, Assistant Engineer of the Proprietors of the Locks and Canals, Lowell, Mass., who was Chairman of the Sub-Committee on "Flood Formulæ for New England" of the Committee on Floods of the Boston Society.

The author further states that the formulas of the Boston Society of Civil Engineers[28] "are based on the assumption that the drainage area involved is rectangular and of uniform slope, like a shingle on a roof, and without any well-defined channels" (see text following Equation (1)). Although it is true that such rectangular drainage areas were used in investigating the effects of different shapes of areas on the hydrograph, the final formula was a general one. The flood coefficients, C_f, were determined and flood hydrographs on a unit basis called "characteristic flood curves" were given for a number of New England rivers. These are quite similar to the distribution graphs of the "unit graph" system developed more than a year later.

Corrections for *Transactions:* The report of the Committee on Floods of the Boston Society of Civil Engineers was published in September, 1930, and not in 1932 as given in the author's Footnote 6. The author devotes a paragraph in criticism of the Boston Society of Civil Engineers formulas. Both of these are quoted incorrectly. The expression $T \propto \sqrt{M}$ should be qualified as applying to similarly shaped drainage areas or as it was given,[28a] $T = C\sqrt{M}$, in which C is a coefficient based on the drainage area characteristics. The formula for flood discharge should be, using the author's notation, $Q_p = C_f \sqrt{M}\, R$, in which R is the total run-off in inches. As given by the author the rainfall I in inches is used instead of R.

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[28] *Journal*, Boston Soc. of Civ. Engrs., September, 1930, p. 297.
[28a] *Loc. cit.*, p. 389.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

DESIGN OF DOWELS IN TRANSVERSE JOINTS OF CONCRETE PAVEMENTS

Discussion

BY W. O. FREMONT, M. AM. SOC. C. E.

W. O. FREMONT,[15] M. AM. SOC. C. E. (by letter).[15a]—Under "Present Construction Practice" the author states that the "exact mathematical solution" of the problem pertaining to a dowel structure of infinite length has been presented by Professor Timoshenko and Mr. Lessels. This "exact mathematical" theory is nothing more than a very approximate first tentative approach to the analysis of the actual behavior of a straight elastic bar of uniform rigidity on an elastic foundation. It is simulated roughly by the same bar, continuously connected to an absolutely rigid support by uniformly distributed elastic columns, of constant length and cross-section and constant elastic characteristics (see Fig. 11). It is assumed that each individual column

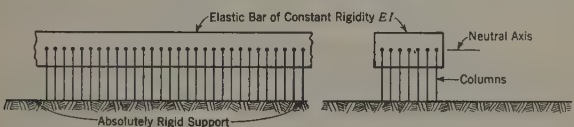

FIG. 11

must lengthen or shorten uniformly and independently, without affecting any of the other columns. This assumption implies that at the upper ends the columns are connected with the neutral plane of the bar by means of hinges. At the lower ends they are fixed to the absolutely rigid support. In this way, the bar is assumed to rest on an ideal brush with an absolutely rigid back, and with ideal bristles extending to, and connected with, the neutral plane of the bar by ideal hinges. It is implied: That no shear or friction acts between the bristles; that they do not bend; that they are of very small cross-section; and, that they are spaced close to one another. Let L = the constant length

NOTE.—This paper by Bengt F. Friberg, Assoc. M. Am. Soc. C. E., was published in November, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1939, by L. E. Grinter, M. Am. Soc. C. E.

[15] Bridge Designing Engr., State Highway Dept., Lansing, Mich.

[15a] Received by the Secretary March 7, 1939.

of each column; A_0 = the cross-section area; E = the modulus of elasticity; F_0 = the force in a column that causes lengthening or shortening; ΔL = the change in length of a column under an external load, F_0; N = number of columns per unit area; Σ_p = the sum of the forces in the uniformly stressed columns, referred to a unit area; and, k = modulus of the elastic foundation. Then,

$$F_0 = \frac{E\,A_0\,\Delta L}{L} \quad \text{(23a)}$$

$$\Sigma_p = N\,F_0 = k\,\Delta L \quad \text{(23b)}$$

$$k = \frac{N\,E\,A_0}{L} \quad \text{(23c)}$$

and, if $N\,A_0 = 1$,

$$k = \frac{E}{L} \quad \text{(23d)}$$

If $E = 4.5 \times 10^6$ lb per sq in. and $L = 4.5$ in., $k = 10^6$ lb-in.$^{-3}$.

The theory based on the foregoing assumptions is also referred to as the elementary theory of beams on elastic foundation. In adapting it to his problem, Mr. Friberg makes a number of assumptions, which the writer proposes to discuss.

Assumption 1.—The paper is predicated on the principle that the concrete surrounding the dowel behaves as is assumed in the elementary theory of beams on elastic foundations. Conditions similar to this ideal exist only in exceptional cases, if at all, in engineering practice. Nevertheless, the theory based on the foregoing assumption yields satisfactory approximations in some cases. Cases are known in which these approximations are not satisfactory. In dowel construction, the columns may be changed to thin plates of similar characteristics perpendicular to the dowel; and one can visualize easily what will happen when the concrete surrounding the dowel, especially near the ends, is suddenly released from all the shear forces interacting between the plates.

The elementary theory depends upon the assumption that Hooke's law holds. Shear tests on doweled concrete block specimens invariably leave permanent deformations. The first run of the test below the "critical point"[16] may produce a shear-deflection curve that is almost a straight line passing through the zero point for the total, elastic, and permanent deflections. The second run shows a considerable deviation from the straight line especially if the residual deformation of the first run is considered. There are plastic deformations that are not considered by the elementary theory. If the dowel is painted or greased the greater compressibility of the film of paint or grease affects the behavior of the system considerably, especially if the paint is destroyed with the passing of time or by friction between the dowel and the concrete.

It is evident that, by the adoption of the elementary theory for doweled pavement slabs, only a very rough approach to the actual conditions is made and one must be prepared for surprising results when derivations on the basis

[16] "Joint Testing Experiments with a Theory of Load Transfer Distribution Along the Length of Joints," by J. W. Kushing and W. O. Fremont, *Proceedings*, Fifteenth Annual Meeting, Highway Research Board, December, 1935.

of this theory are compared with actual observations of construction work or test specimens.

Values of dowel shears in concrete blocks, at the point of failure, computed on the basis of the elementary theory, varied between 4 000 and 14 000 lb (350%) in one set of tests. In the case of dowels of circular cross-section there is a further complication as the distribution of pressures upon the concrete may be different from the one in the case of rectangular prismatic bars.

Assumption 2.—Under "Application of Design Formulas" Mr. Friberg states: "In practice, the dowels are usually not very stiff compared with the concrete pavement. It is an acceptable approximation, then, to presume that a point of contraflexure exists in the dowel at the center of the joint, * * *." By a method similar to the one proposed by Dean Westergaard[17] and, by the introduction of the elementary theory discussed herein under "Assumption 1," the writer has made the following observations:

Load at the joint, in pounds	9 000
Thickness of the slab, in inches	7
Modulus of elasticity of concrete, in pounds per square inch	4.5×10^6
Joint opening, in inches	1
Diameter of dowels, in inches	$\frac{3}{4}$
Spacing of dowels, in inches, center to center	12
Subgrade modulus (bulk modulus of elasticity) in pounds per cubic inch	250
Shear force, T_0, in pounds	920
Moment (see Fig. 12), in inch-pounds, at the:	
Unloaded face, $M =$	− 369
Loaded face, $M_f =$	551
Center, $M_c =$	91
The relative stiffness, $\beta =$	0.793

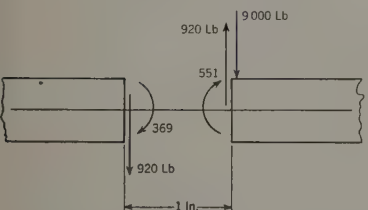

FIG. 12

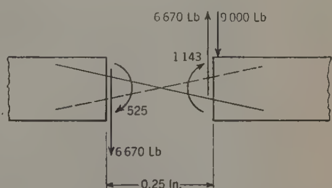

FIG. 13

From the foregoing, the maximum pressure on the concrete, as computed by the formula

$$s = 10^6 \frac{T + \beta\, M_f}{2\, \beta^3\, E\, I} \qquad (24)$$

s $\frac{920}{474} \times 10^3 + \frac{71.3}{474} \times 10^3 + \frac{364}{474} \times 10^3 = 2\,867$ lb per sq in.

[17] "Spacing of Dowels," by H. M. Westergaard, *Proceedings*, Eighth Annual Meeting, Highway Research Board, 1928.

Of this pressure, 150 lb per sq in. (that is, 5.25% of the total pressure) is due to the moment $M_c = 91$ in-lb. If the misalignment is uniformly 0.5 in. per foot of dowel, but alternating in opposite direction and symmetric with respect to the load, the condition shown in Fig. 13 was obtained for the center dowel when the opening was decreased to 0.25 in. by expansion. In this case,

$$M_c = 6\,670 \times 0.125 - 525 = 308 \text{ in-lb.; and, } s = \frac{6\,670}{474} \times 10^3 + \frac{244}{474} \times 10^3 + \frac{906}{474} \times 10^3 = 14\,100 + 515 + 1\,915 = 16\,530 \text{ lb per sq in.,}$$

in which 515 is due to M_c and is 3.1% of 16 530.

Assumption 3.—A third assumption involved in the paper is that the foundation modulus of the concrete is assumed to be $k_c = 10^6$ lb per cu in.

Assumption 4.—Under "Design Formulas for the Ordinary Case," the author states: "Under most construction conditions the bearing stress on the concrete is the critical one."

Tests by the writer indicate that the concrete beneath the dowel is subject to strains corresponding to very high compression stresses. According to the mathematical theory of elasticity these stresses must decrease in the direction away from the dowel, and therefore the destruction or plastic deformation of the concrete is limited to a region in the immediate vicinity of the dowel. If the dowel is near the neutral plane of the pavement slab this plastic deformation has no further effect on the slab itself.[18]

It has not been demonstrated that the plastic deformation around the dowel has ever impaired the strength of the pavement slabs directly, but such deformation impairs the load transfer and therefore increases the extreme fiber stresses in the slabs and decreases the strength of the structure indirectly. Therefore, as long as the fiber stresses are under control the pressure of the dowel on the concrete, with any resulting plastic deformation, should not be considered "critical."

Assumption 5.—In the sentence preceding "Dowel Deflection Across a Joint" Mr. Friberg states that, under the conditions cited, the dowels could be made as much as 25% shorter without appreciably affecting the maximum

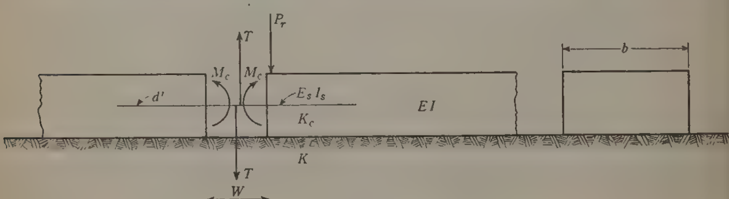

FIG. 14

stresses in and around the dowels. If the plastic deformation is taken into consideration, as well as the effects of paint and grease, the points of stress reversal may be found to have moved deeper into the concrete. Furthermore,

[18] "Plasticity," by A. Nadai, McGraw-Hill Book Co., Inc., 1931.

the use of caps reduces the effective length of the dowels. Therefore, it is necessary to consider, carefully, all the factors affecting the length of the dowels.

Assumption 6.—In the sentence preceding "Effects of Dowel Spacing on Stress Relief" appears the statement: "Therefore, the effect of pavement slope upon dowel deflection across joints may be neglected safely." Consider two semi-infinite, abutting, beams on an elastic foundation, connected by a dowel with a load P_r at the edge of one of them, as shown in Fig. 14:

Let

$$\Delta = 6\,n\,[1 + (1 + \beta\,W)^2] + 6\,m^3\,[1 + (1 + \epsilon\,W)^2] + \beta^3\,W^3 \quad \text{(25a)}$$

$$n = \frac{E_s\,I_s}{E\,I} \quad \text{(25b)}$$

$$\beta = \epsilon\,m = \sqrt[4]{\frac{k\,b}{4\,E\,I}} \quad \text{(25c)}$$

and,

$$\epsilon = \sqrt[4]{\frac{k_c\,d'}{4\,E_s\,I_s}} \quad \text{(25d)}$$

Then,

$$T = \frac{3\,n\,(2 + \beta\,W)}{\Delta}\,P_r \quad \text{(26a)}$$

and,

$$M_c = \frac{n\,P_r}{2\,\beta\,[2\,(m + n) + \beta\,W]} \quad \text{(26b)}$$

On the other hand, the static and elastic states of a dowel in a pavement may be defined by the angular displacements α_T and α_{T_r} of the abutting slabs at the faces of the joint and their relative vertical deflection Δ_v, independently of the vertical forces and moments acting on the slabs. Then the bending moment at the center of the dowel will be:

$$M_c = \frac{\epsilon\,E\,I\,(\alpha_T + \alpha_{T_r})}{2 + \beta\,W} \quad \text{(27a)}$$

and, the shear forces T in the dowel will be:

$$T = \frac{6\,\epsilon^3\,E\,I\,[2\,\Delta_v + W\,(\alpha_{T_r} - \alpha_T)]}{6\,[1 + (1 + \beta\,W)^2] + \beta^3\,W^3} \quad \text{(27b)}$$

in which d' is the diameter of the dowel; k_c is the foundation modulus of concrete; and, k is the foundation modulus of subgrade.

Equations (26) and (27) were developed on the basis of the elementary theory of beams on an elastic foundation, for the slabs and for the dowels.

The moment M_c does not depend on the relative deflection Δ_v, but depends only on the angular displacements α_T and α_{T_r}—namely, their sum, to which it is proportional. If $\alpha_T + \alpha_{T_r} = 0$, $M_c = 0$, and *vice versa*. If $W\,(\alpha_{T_r} - \alpha_T)$ is small as compared with $2\,\Delta_v$ then T depends practically only on Δ_v and is proportional to it.

The angular displacements α_T, α_{T_r}, and the relative deflection Δ_v were computed for the case of two abutting beams 7 in. by 12 in., connected by a

$\frac{3}{4}$-in. dowel having a joint opening of 1 in. and loaded at one edge by $P_r = 1\,000$ lb, on a subgrade for which $k = 250$ lb per cu in. and $k_c = 10^6$ lb per cu in. By Equation (26) it was found that $\Delta_v = (10.47 - 6.96)\ 10^{-3} = 3.48 \times 10^{-3}$ in.; $\alpha_T = 187 \times 10^{-6}$; and, $\alpha_{T_r} = 274 \times 10^{-6}$. Substituting these values in Equation (27), $T = 384 - 4.8 = 379.2$ lb.

The quantity 4.8 is that part of T due to the angular displacements α_T and α_{T_r} and constitutes only 1.25% of T.

Assumption 7.—From the Rule stated by the author under "Effects of Dowel Spacing on Stress Relief" it follows that the distribution of shear among the dowels is independent of the elastic characteristics of the joint construction and the amount of joint opening. The reasoning presented in the text preceding the Rule is not convincing and is incomplete. The shear considered near the points of maximum negative moments will be nearly zero only for points near the very edge. For the consideration of equilibrium, including the dowel shear forces, an element of finite dimensions must be considered. For such an element the shears referred to will not be zero on the basis of the reasoning presented in the paper.

For absolutely stiff, or rigid, joint units 50 % of the load will be transferred by one unit at the point of application of the load. For very elastic (say rubber units) the load transfer curve will approach the elastic curve of the loaded slab. In the latter case no load transfer will occur, and almost the entire load will be supported by the loaded slab alone.[19] In the case of dowels in concrete slabs the engineer will be unable even to approach a condition of absolute rigidity in joint construction. The characteristic condition, rather, would tend toward the other extreme.

Assumption 7 calls for a triangular load distribution. In the case of rigid joints there would not be any definable distribution. In the case of very flexible joints the distribution defined by the deflection curve will extend for a distance equal to more than $\pi \times l$ on both sides of the load; at $\pi \times l$ the deflections are still greater than 10% of those under the load.[20]

Field tests on doweled pavements (8-in. slabs, 11 ft wide, loaded on the edge at the center of the pavement, with a 9 000-lb load, 13 dowels of $\frac{3}{4}$-in. diameter, spaced 12 in.) produced edge-deflection curves for the loaded and unloaded edges extending the entire width of the pavement which did not intersect at any point. The relative deflections of the two slabs at the outer edges were still 11% and 15.6% of those under the load.

The radius of relative stiffness was computed as $l = \sqrt[4]{\frac{4.5 \times 10^6 \times 8^3}{12 \times 161}}$ $= 33$ in. This shows that dowels beyond the point 1.8 l may still have quite a noticeable influence upon the bending moment under the load and the corresponding extreme fiber stresses. The load transfer distribution depends to a great extent on the rigidity of the joint construction.

[19] "Joint Testing Experiments with a Theory of Load Transfer Distribution Along the Length of Joints," by J. W. Kushing and W. O. Fremont, *Proceedings*, Fifteenth Annual Meeting, Highway Research Board, December, 1935, p. 154.

[20] "Computation of Stresses in Concrete Roads," by H. M. Westergaard, *Proceedings*, Fifth Annual Meeting, Highway Research Board, December 3–4, 1925, Fig. 9.

Assumption 8.—Equation (16) is presented as the expression for defining the shear on a dowel directly under the load. When so defined, P is considered to be quite independent of the dowel spacing and the action of all the other dowels in the joint except the one directly under the load. Consider a pave-

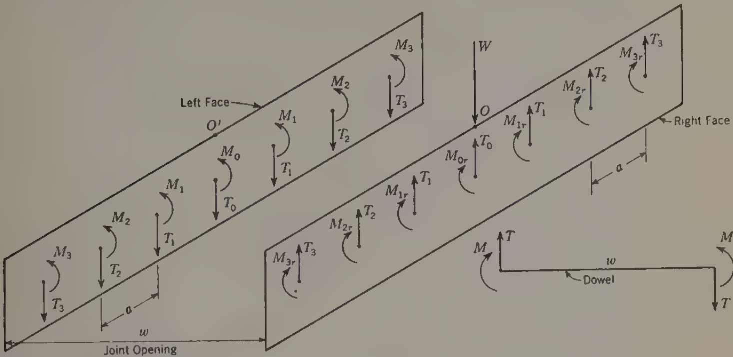

FIG. 15

ment construction consisting of two abutting slabs, connected by joint units, on an elastic foundation, with one dowel directly under the load W in Fig. 15.

Equalizing the deflections (Fig. 15):

$$(W - T_0)\, y_p - 2\Delta_T + (M_{0r} - M_0)\, y_m + y_m' - y_m'' = T_0\, y_p + T_0\, y_d \quad \ldots\ldots (28)$$

in which (neglecting the effect of M_0 on y_d):

$$y_m' = a_{11}\, M_1 + a_{12}\, M_2 + \cdots \quad \ldots\ldots (29a)$$

and

$$y_m'' = y_m' + W\,(a_{11}\, T_1 + a_{12}\, T_2 + \cdots) \quad \ldots\ldots (29b)$$

Substituting Equation (29) in Equation (28):

$$T_0 = \frac{W\, y_p - 2\,\Delta_T + [(M_{0r} - M_0)\, y_m - W\,(a_{11}\, T_1 + a_{12}\, T_2 + \cdots)]}{2\, y_p + y_d} \quad \ldots (30)$$

Ignoring the influence of the bending moments:

$$\frac{T_0}{W} = \frac{1 - 2\,\dfrac{\Delta_T}{W\, y_p}}{2 + \dfrac{y_d}{y_p}} \quad \ldots\ldots (31)$$

in which,

$$\Delta_T = 2\, A_1\, T_1 + 2\, A_2\, T_2 + \cdots \quad \ldots\ldots (32)$$

is the deflection at Points O and O' produced by the shears T_1, T_2, $\cdots$ alone; and, $W\, y_p$ is the deflection produced at Point O from Load W alone. It is clear that the doubled ratio $\dfrac{\Delta_T}{W\, y_p}$ cannot be neglected in Equation (31) without introducing considerable error.

Consider a 7-in. slab with $\frac{3}{4}$-in. dowels. The foundation is elastic, with a foundation modulus of 250 lb per cu in.; the joint is open 1 in.; five dowels are spaced 12 in. apart; and, a load of $W = 9\,000$ lb is concentrated over the center dowel. By a method similar to the one proposed by Dean Westergaard[17] and the introduction of the elementary theory described herein under Assumption 1, the writer has determined the following values: $T_0 = 920$ lb; $T_1 = 673$ lb; $T_2 = 480$ lb; $\Delta_T = 4.086 \times 10^{-6} \times 673 + 330 \times 10^{-6} \times 480 = 4.34 \times 10^{-3}$; $y_p\, W = 2.35 \times 10^{-6} \times 9\,000 = 21.18 \times 10^{-3}$; and (according to Table 2 of the paper), $y_d = 0.0094 \times 10^{-3}$. Consequently, by Equation (31):

$$T_0 = 9\,000 \frac{1 - 2\dfrac{4.34}{21.18}}{2 + \dfrac{94}{23.5}} = 885 \text{ lb},$$

which is not seriously different from the value $T_0 = 920$ lb determined by the writer previously. Equation (16) of the paper yields:

$$T_0 = 9\,000 \frac{1}{2 + \dfrac{94}{25}} = 1\,562 \text{ lb},$$

an increase of 70%. Mr. Friberg's distribution, as affected by Assumption 7, yields:

$$T_1 = \frac{1.8\, l - a}{1.8\, l}\, T_0 \qquad (33a)$$

$$T_2 = \frac{1.8\, l - 2\, a}{1.8\, l}\, T_0 \qquad (33b)$$

and

$$T_k = \frac{1.8\, l - k\, a}{1.8\, l}\, T_0 \qquad (33c)$$

From Equation (32):

$$\Delta_T = 2\, T_0 \sum_{k=1}^{k=m} \left(1 - \frac{a\, k}{1.8\, l}\right) A_k \qquad (34)$$

in which $m + 1 > \dfrac{1.8\, l}{a} > m$; and, m is an integer. Substituting Equation (34) in Equation (31):

$$\frac{T_0}{W} = \frac{1}{2 + \dfrac{y_d}{y_p} + 4\dfrac{\sum_{k=1}^{k=m} A_k - \dfrac{a}{1.8\, l}\sum_{k=1}^{k=m} k\, A_k}{y_p}} \qquad (35)$$

in which values of A_k are read from curves presented by Dean Westergaard.[17] In the case of the present numerical example, the conditions stated for Equation

(34) are satisfied when $l = 26.8$ in. and $m = 4$. Then: $A_0 = 2.35 \times 10^{-6} = y_p$; $A_1 = 2.043 \times 10^{-6}$; $A_2 = 1.650 \times 10^{-6}$; $A_3 = 1.300 \times 10^{-6}$; $A_4 = 0.935 \times 10^{-6}$; $\sum_{k=1}^{k=4} A_k = 5.928 \times 10^{-6}$; $\sum_{k=1}^{k=4} k\, A_k = 12.983 \times 10^{-6}$; and, $\sum_{k=1}^{k=4} A_k - \frac{12}{1.8\, l} \sum_{k=1}^{k=4} k\, A_k$ $= 2.682 \times 10^{-6}$. Therefore, by Equation (35),

$$T_0 = \frac{9\,000}{2 + \frac{94}{23.5} + 4\,\frac{2.682}{2.35}} = 871 \text{ lb}$$

which checks, reasonably well, the results derived by other methods. Assumption 8, as expressed by Equation (16), should be revised to conform to Equation (35).

The maximum fiber stress at Point O, Fig. 15, may be expressed as

$$s = s_W - s_0 - 2\, s_T \qquad (36)$$

in which s_W is the component of stress due to W; s_0 is the component due to T_0; s_T is the component due to T_1, T_2, etc., on one side of the load; and,

$$\gamma_0 = \frac{s_0 + 2\, s_T}{s_0} \qquad (37)$$

represents the amount of total stress relief, expressed in the partial relief, produced by the dowel directly under the load as a unit.

Then, the total stress relief at Point O, expressed in terms of the stress s_W, corresponding to the state in which there is no relief (free ledge) as a unit, will be

$$\gamma = \frac{s_0 + 2\, s_T}{s_W} = \frac{s_0}{s_W}\, \gamma_0 \qquad (38a)$$

Since $s_W = u\, W$; and, $s_0 = u\, T_0$:

$$\gamma = \frac{T_0}{W}\, \gamma_0 \qquad (38b)$$

and—since $M_0 = m_{0,0}\, T_0$; $M_T = \sum_{k=1}^{k=m} m_{0,k}\, T_k$; $s_0 = \frac{M_0}{W_0}$; and, $s_T = \frac{M_T}{W_0}$—

$$\gamma_0 = 1 + 2 \sum_{k=1}^{k=m} \frac{m_{0,k}}{m_{0,0}} \frac{T_k}{T_0} \qquad (38c)$$

in which $m_{0,k}$ are picked from the curves presented by Dean Westergaard.[17]

Formulas similar to Equations (31), (35), (37), (38*b*), and (38*c*) must be derived for the case in which Load W is placed between two dowels.

To compute γ one must know $\frac{T_0}{W}$, Equation (38*b*), in which T_0 is the shear in the dowel directly under the load, with all the other dowels acting according to Assumption 7 and not as assumed by Mr. Friberg in the derivation of Equation (16). For T_0 Mr. Friberg takes the shear in the dowel directly under

the load, on the assumption that the other dowels are inactive. Therefore, T_0 in Equation (38*b*) must be determined according to Equation (35). Equation (38*c*) can be computed conveniently by means of graphs.

In introducing Table 3 the author states that the unit moments were taken directly from data presented by Dean Westergaard[7]; and, again, in discussing Fig. 6, he states that the curves were computed in accordance with methods established by Dean Westergaard. The values of y_p may also be obtained from Dean Westergaard's diagrams. Would it not be more logical to use these diagrams throughout the entire analysis, and then compare the theoretical results with experimental measurements, rather than mix theory with experiment?

The percentages of load transfer given in Table 4, the computed values of stress relief in Table 5, and all conclusions drawn on the basis of these tables, need to be adjusted to Equations (35), (38*b*), and (38*c*).

Assumption 9.—The effect of vertical dowel misalignment is determined by Equation (18) on the assumption of uniform alternate misalignment. Under such conditions the maximum fiber stresses in the slab are affected very little. Such ideal conditions of misalignment will occur seldom in actual pavements.

By the same method (with a 7-in. pavement and five dowels spaced at 12 in., the center dowel being misaligned 1 in. per ft of dowel; the original opening was 1 in. and the final was zero), the writer has found: $T_0 = 10\,740$ lb; $T_1 = -3\,380$ lb; and, $T_2 = -2\,025$ lb. The corresponding fiber stresses in the slab were found to be: $s = -10\,740 \times 0.0364 + 6\,760 \times 0.0123 - 4\,050 \times 0.00306 = -321.4$ lb per sq in. For smaller misalignments this stress will be proportionately smaller, being zero for zero misalignment and -80.4 lb per sq in. for $\frac{1}{4}$ in. misalignment. The actual misalignment stresses will be smaller owing to the relief afforded by plastic deformation and by the paint or grease effect, which must also be taken into consideration.

The pressure between dowel and concrete is not critical as long as the fiber stresses are under control.

The Nature of Assumptions.—In making assumptions for the simplification of problems in applied mechanics it is important to know the kind and amount of their effect upon the result. Another suggestion is to vary the assumed values so that, by definite variations, a successively closer and closer approach can be made to the exact solution.

Assumptions affecting the laws of mechanics, or conclusions derived from such laws, do not always indicate, directly and clearly, the effect they have upon the results without laborious investigation. Assumptions affecting the characteristics of the materials used are mostly of the same nature (as, for example, the assumption of the linear, or Hooke's, law instead of some curvilinear law). Assumptions affecting the general make-up of the system, without any sacrifice in the exactness of fundamental laws or the characteristics of the materials, often afford a much clearer and easier appraisal of the errors involved by using the approximate solution instead of the exact one. For instance, in the case of a doweled pavement slab construction on an elastic foundation the problem

[7] "Computation of Stresses in Concrete Roads," by H. M. Westergaard, *Proceedings*, Fifth Annual Meeting, Highway Research Board, 1925.

may be simplified by considering only one dowel under the load. It is clear, then, that by introducing three dowels instead of one, the designer approaches closer to the exact solution. With any one of such assumptions (1 or 3 or 5, dowels) he has the advantage of knowing "where he stands"; the road to closer solutions is clear and open and the preceding step helps to make the following one.

Conclusion.—The writer is of the opinion that the method of dowel action analysis introduced by Dean Westergaard,[17] supplemented and extended either by the introduction of the elementary theory of beams on elastic foundation (described herein as Assumption 1) and the consideration of plastic deformation and the effect of grease or paint, or by the use of experimental curves obtained in the laboratory with specimens of actual joint units, is the basic method. On the basis of the data so obtained convenient graphs can be prepared for the use of the practical and field engineer.

Any other methods may be used within definite limits, for definite joint constructions, if their agreement with the basic method has been established within these limits. This is especially important for the analysis of new types of joint construction.

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DISCUSSIONS

SETTLEMENT STUDIES OF STRUCTURES IN EGYPT

Discussion

BY D. P. KRYNINE, M. AM. SOC. C. E.

D. P. KRYNINE,[17] M. AM. SOC. C. E. (by letter).[17a]—The importance of settlement observations on full-sized structures has often been emphasized in technical literature. The paper by Professor Tschebotareff is a valuable contribution in this respect.

Two Cases of Settlement Discussed.—Before examining the interesting collection of settlement contour lines presented by Professor Tschebotareff, the writer wishes to express some general thoughts on the shape of such lines. Two different cases will be distinguished: Case A, in which the structure, or the tips of the piles supporting it, reaches the top (or nearly the top) of a compressible layer; and Case B, in which the compressible layer is at a considerable depth. Most of the structures discussed in the paper are examples of Case A.

Settlement in Case A.—When the compressible layer is more or less uniform and is sufficiently thick, it appears first that it may be considered as a semi-infinite, elastically isotropic body. In such a case settlements could be computed according to elastic theories:

$$s_0 = \frac{P}{r}\frac{1 - \mu^2}{\pi E} \qquad (2)$$

in which: s_0 = settlement at a given point; P = load; r = horizontal distance from the load P to the given point; μ = Poisson's ratio; and E = modulus of elasticity of the earth material.

However, it is difficult, and perhaps impossible, to determine the elastic constants of the given earth material in the field. Furthermore, the settlement of a clay deposit is due also to the load squeezing moisture from its pores so that the actual settlement is many times greater than the computed value.

NOTE.—This paper by Gregory P. Tschebotareff, M. Am. Soc. C. E., was published in October, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1939, by Messrs. L. C. Wilcoxen, Trent R. Dames, and Edwin J. Beugler; and May, 1939, by W. S. Hanna, Esq.

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[17a] Received by the Secretary April 20, 1939.

An assumption can be made, however, which is of the same order of accuracy as assumptions made in the theory of consolidation; namely, that the actual value of the settlement to be expected is proportional to the value of s_0, as computed using Equation (2). In other words, the settlement is proportional to the ratio $\frac{P}{r}$. In Equation (2) substituting

$$C = \frac{1 - \mu^2}{\pi E} \qquad (3)$$

relative values of the expected settlement can be computed easily. Let s_0' and s_0'' represent settlements at two different points of the ground surface, as caused by load P; and, let r' and r'' represent respective horizontal distances of these points from load P. Then:

$$s_0' = C \frac{P}{r'} \qquad (4a)$$

and

$$s_0'' = C \frac{P}{r''} \qquad (4b)$$

from which:

$$s_0' : s_0'' = r'' : r' \qquad (5)$$

that is, elastic settlements at two different points of the ground surface are inversely proportional to their horizontal distances from the load. Relative values of elastic settlement at any point of the ground surface, as caused by a uniformly loaded area of arbitrary shape, can be computed, even in complicated cases, by using the method of graphic integration. W. Steinbrenner has given analytical formulas and has prepared tables of such settlements in the simplest case of rectangular loaded areas.[18]

It should be noted, however, that Professor Terzaghi advises that the settlement of shallow layers (such as described under "Case A") be computed in the same manner as that of deep layers[19]; and Professor Tschebotareff follows that procedure.

Settlement in Case B.—Should the compressible layer occur at a considerable depth, its consolidation is attributed to the action of the vertical pressure, p_z, which is computed using the well-known Boussinesq formula. The distribution of the vertical pressure at the top surface, $a\ a_0\ a'$, of the compressible layer is given in the form of a curve, $a\ a_0\ a'$, in Fig. 15. A similar distribution occurs along any other horizontal plane of the compressible layer; as, for instance, at the bottom (Section $b\ b'$, Fig. 15). All ordinates of the distribution curve traced at the bottom of the layer would be smaller than those at the top. Consider a vertical section ($o\ r\ u\ m\ n$, Fig. 15) of the earth mass. Plotting vertical pressures, p_z, at different depths of this section as horizontal ordinates, a curve, $o\ s\ w\ p\ q$, would be obtained. The shaded area, $m\ p\ q\ n$, multiplied by a coefficient depending on the compressibility of the layer in question

[18] "Tafeln zur Setzungsberechung," by W. Steinbrenner, *Die Strasse*, Vol. 1 (1934), p. 121.

[19] "Theorie der Setzungen von Tonschichten," by Charles Terzaghi and O. K. Fröhlich, Leipzig and Vienna (1936), p. 14.

("specific loss of moisture" as discussed hereafter), furnishes the value of the final settlement at Point *o*. It is assumed in this connection that a column of earth such as *o m* or *t t'* settles independently of the remainder of the earth mass, and that, in sinking down, it does not undergo any change of volume or shape. In other words, soil deformations above the compressible layer *a a' b' b*, Fig. 15, are neglected. The soil above the compressible layer is thus assumed to be deprived of shearing resistance.

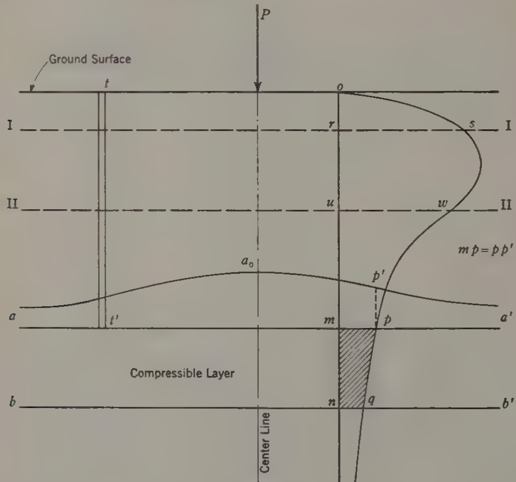

FIG. 15

It should be noted that the Boussinesq formula for the vertical pressure does not contain the elastic constants E and μ; hence, it is valid for any homogeneous, elastically isotropic material. Were the entire mass made of clay, or of steel, or of some other homogeneous material, which could be considered as elastically isotropic, Boussinesq's formula for the vertical pressure would furnish fairly satisfactory results, regardless of the type of material. Its use becomes questionable, however, in the case of a compressible layer, since there is discontinuity at the top of the latter, and the material is no longer homogeneous.[20] This phenomenon has been explained also by the "bridging (or arching) effect" of a sand or gravel layer which overlies a compressible clay layer. According to the latter view, the load applied at the earth's surface is spread by the intermediate sand or gravel layer so that the magnitude of the stresses and their distribution in the lower compressible layer are thus modified.

Settlement at a Considerable Depth as Caused by a Loaded Area.—According to Saint Venant's principle, stresses at a point within a body remote from the surface where a system of forces is applied are practically the same as those caused by the resultant of that system. Hence, stresses and strains in a deep

[20] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 885.

compressible layer are practically the same as those caused by the resultant P of the forces acting on that area. It is evident that stresses thrown to the compressible layer in question should be practically symmetrical with respect to the vertical line of action of force, P. In other words, the set of lines of equal settlements at the earth's surface should approach a set of concentric circles if the seat of settlement is at a considerable depth. Professor Tschebotareff's contour lines are far from being concentric circles, however; this is because, in reality, the settlements in question have their seats in rather shallow layers.

Influence of the Rigidity of the Superstructure.—Professor Tschebotareff presents interesting comparisons between theoretical settlements in the case of rigid and non-rigid (flexible) superstructure, or more accurately, of the rigid and non-rigid base of the superstructure. Quite striking is the case of Fig. 1 in which lines of actual settlement follow rather closely the lines of equal pressure shown in that figure, which is assumed to be rigid. It is important to notice that, in this case, the seat of settlement is in a rather shallow layer. The writer believes that the influence of rigidity on the pressure distribution at a horizontal plane decreases with the depth. This is in accord with Saint Venant's principle since at a greater depth the values of the stresses are controlled merely by the total weight of the entire structure (assuming that this weight is the same in the case of both rigid and non-rigid superstructure). In the same degree, the size of a rigid structure may have a certain influence on the settlement; apparently, the larger the structure, the less will be the influence of the rigidity on the stresses underneath the central part of that structure. In the limiting case in which both rigid and non-rigid structures are infinitely long and infinitely wide, there should be no difference whatsoever between a rigid and a non-rigid structure, since in such a case the value of the settlement would depend entirely on the properties of the earth material.

Depth at Which the Vertical Pressure Should Be Computed.—Professor Tschebotareff computes the vertical pressure "at approximately two-thirds the depth of the compressible layer" (in some occasions one-half that depth).

This approximation is not always warranted. In the case of a deep layer, such as that bounded by Plane $a\ a'$ (Fig. 15) and Plane $b\ b'$, the area controlling the settlement is $m\ p\ q\ n$, and its average ordinate is approximately at the middle of the layer. Should the layer be close to the earth's surface (for instance, the layer bounded by Planes I–I and II–II) the situation is entirely different; and the average ordinate may or may not be at the middle or at two-thirds of the height of area $r\ s\ w\ u$. Professor Tschebotareff's assumption is practically correct in the case of a shallow layer between the ground surface and Plane I–I (Fig. 15).

Coefficient of Compressibility, X.—Professor Tschebotareff is to be commended for introducing the coefficient of compressibility, X, as defined by Equation (1), and in comparing the laboratory value of this coefficient with the corresponding field value. Since the value of $\frac{a}{1 + e'}$ in Equation (1) is the reciprocal of what is analogous to the modulus of elasticity, that value is measured by a reciprocal of stress (in square centimeters per gram, as Professor Tschebotareff states). The second part of Equation (1) proves it, since

$\frac{s}{d'}$ is a strain and Δp is a stress, so that X is a strain-stress ratio. For practical purposes it is convenient, however, to express X in units of length as is done by the author as well as by Terzaghi and Fröhlich.[21] The latter expresses the value of $v = \frac{a}{1 + e'}$ as "specific loss of moisture." If a prism of arbitrary horizontal cross-section is imagined to be cut from a compressible layer 1 m thick, and is still confined laterally and subjected to the action of a compressive stress of 1 gram per cm², v will be the height (in meters) of the column of water squeezed from that prism. If the stress is increased 1 000 times and equals 1 kg per cm² (1 ton per sq ft, approximately), the value of v is to be multiplied by 1 000, thus becoming Professor Tschebotareff's coefficient:

$$X = \frac{a}{1 + e'} \times 1\,000 \qquad (6)$$

The value of X is still in meters; for instance, $X = 0.010$ m, as in Professor Tschebotareff's example tabulated in the text following Equation (1). The expression "specific loss of moisture" has been introduced as an analogy of the conception "specific heat" in physics and thermodynamics.

Quite ingeniously, Professor Tschebotareff has shown that it is not necessary to make a complete consolidation test if an error of 3% or 4% is tolerable. He thus obtains the valuable part of the test without wasting time and energy on non-essential phases of it.

Conclusions.—In the five years, 1934 to 1939, the literature on soil mechanics has been substantially increased by observations of settlement in different parts of the world. To the original observations in Western Europe and the eastern sections of the United States, observations in the Western United States, in the South (Texas and Louisiana), in Mexico, and now in Egypt, are added. Numerous interesting observations in Russia have been published in that language. All this material needs to be digested and classified. The writer believes that a new chapter in the study of soil mechanics is opening which may be called "geography of settlement observations," and that its adequate development is of great importance for foundation engineers.

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[21] "Theorie der Setzungen von Tonschichten," by Charles Terzaghi and O. K. Fröhlich, Leipzig and Vienna (1936), p. 24 *et seq.*

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

THE YELLOW RIVER PROBLEM

Discussion

BY MESSRS. C. S. JARVIS, AND E. W. LANE

C. S. JARVIS,[27] M. AM. SOC. C. E. (by letter).[27a]—Many essential data hitherto unavailable to the engineering profession have been presented by the authors, and they have suggested such remedial measures as seem to have most practical value, according to their intimate knowledge of local conditions prevailing throughout the Yellow River Basin.

From the basic information and also from the recommendations thus presented, it appears that the most dependable and positive relief from floods would entail a combination of head-water storage and detention works, extensive land surface treatment for the express purpose of soil and water conservation, and such bank protection, flood spillways, and by-pass facilities as would best afford the needed channel capacity and stability, together with the desired security for inhabitants and their properties. Moreover, there is assurance that the most critical sections of protective works may be located and strengthened well in advance of the flood stages, instead of waiting until the danger is imminent or actually upon them, according to the ancient fatalistic practice.

It is interesting to note that the total Yellow River drainage area is nearly twice that of the Ohio, yet its maximum estimated discharge (900 000 cu ft per sec) was only about one-half of that recorded in the Ohio River (1 600 000 cu ft per sec at Paducah, Ky., in April, 1913, and 1 850 000 cu ft per sec at Metropolis, Ill., in February, 1937). Furthermore, the Yangtzekiang drainage area and maximum discharges both compare fairly closely with those of the Mississippi River.

The authors state (in the text preceding "Hydrological Considerations") that the Wei Ho (drainage area, 56 000 sq miles) is almost as important a flood and silt carrier as the Yellow River itself. Although it is less important as a

NOTE.—This paper by O. J. Todd, M. Am. Soc. C. E., and S. Eliassen, Assoc. M. Am. Soc. C. E., was published in December, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1939, by Messrs. J. W. Beardsley, and Elliott J. Dent; and April, 1939, by Messrs. Herbert Chatley, and H. van der Veen.

[27] Hydr. Engr., SCS, Dept. of Agriculture, Washington, D. C.

[27a] Received by the Secretary April 27, 1939.

low-water feeder, it appears that the critical areas awaiting corrective and regulatory measures must be within this tributary basin; and these main flood-source areas may constitute only a minor portion. Detention and storage facilities would be capable not only of reducing damaging flood crests, but also of augmenting the low-water flow as a natural consequence.

The use of auxiliary flood channels and lateral waterways running nearly parallel to the river course, and supplied by controlled sluices through the embankments, with gateway sills set low enough to divert a portion of the bed-load as well as some of the suspended material along with the released flood water, might go far toward maintaining channel capacities through the removal of bars, or by preventing their formation. At the same time, such sluicing might contribute toward filling in the depressions and undrained areas adjacent to both the irrigable tracts and also the alluvial ridge along which the river now wends its way, continually chafing under the restraints imposed by barrier embankments, and searching for some means of escape to the neighboring low-lands.

E. W. Lane,[28] M. Am. Soc. C. E. (by letter).[28a]—Probably no engineering project under serious consideration would do more to relieve human suffering than the control of the Yellow River in China. In their work on this problem, in the field, the authors have already made major contributions, and this paper, by preserving through times of disorder the most important facts and engineering data previously collected, may well be another major contribution to the success of this project. Small amounts of information on the Yellow River are widely scattered through engineering literature, but in no other place can one find a wealth of material at all comparable with that presented by the authors.

The Yellow River is not only a stream of water, but it is also a stream of sediment, and any engineering plan which will be more than a temporary measure must take this into account. The Yellow River is not unique in this respect—the same thing is true of most rivers. The writer believes that a great deal of trouble will occur in the future because sufficient provision has not been made in a number of engineering projects for the flow of sediment as well as the flow of water. However, the Yellow River is unusual in that the quantity of sediment carried is so great that the necessity of a consideration of it is more readily apparent. In some other streams the action will be the same but it will go on at so slow a rate that the results will be apparent only after many years.

A comprehensive solution for the flood problem of the Yellow River, therefore, must be complete both from the standpoint of the water stream and also from that of the sediment stream. From the practical standpoint, the best plan may be to undertake first only the control of the water stream, since such a procedure may give the greatest return on the money which can be raised in the early years for carrying out the project; but in order to select the best plan it is necessary to view the whole problem and weigh all the disadvantages of tem-

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[28a] Received by the Secretary May 16, 1939.

porarily neglecting the control of the sediment. Some of these disadvantages may not be readily apparent without a careful study of the control of the sediment stream, but may prove to be so great that an apparently satisfactory solution from the limited viewpoint may turn out to be much less desirable than one which, without considering the sediment aspects, appears relatively unpromising.

In order to appraise correctly the problems due to the sediment stream, it is necessary that the laws governing the transportation of sediment be known. Unfortunately knowledge of these laws is very imperfect. However, rapid progress is being made in this field, and it is confidently believed that before long it will be possible to handle problems of sediment quantitatively as engineers now handle those of water. At least approximate solutions of bed-load problems have been available for some time, and quantitative solutions of suspended load relations are being developed. One of the greatest needs for a solution of the Yellow River problem is a further working out of these laws. It is probable that the difficulties of controlling the sediment will be as great or even greater than those of controlling the water, and the need for a knowledge of the laws governing sediment will be as essential to a proper solution as a knowledge of the laws controlling the flow of water.

If the Yellow River were only a river of water, the data presented in this paper indicate that its flood problem probably could be solved with comparative ease by retarding basins. Because of the extremely short duration of the flood peak, the conditions are unusually favorable to retarding basin control. The presence of the heavy sediment load, however, seriously complicates the situation, and in any flood-control plan it must be given most serious consideration.

As shown by the profile given by Colonel Dent,[28b] there is a distinct flattening of the gradient of the river where it leaves the gorge section at Mengtsin and starts across the plain. This point is just below the region in which the most promising storage site appears to be located. The slope in the bottom of the reservoirs would be over six times that just below Mengtsin. As the writer has explained previously,[29] the water flowing across the sediment deposited in the bed of the flood-control basin would pick up a much greater load of sediment than it could carry across the plain, and therefore extensive deposit would take place down stream from the reservoirs, where the slope is flatter. This deposit would fill up the bed of the stream and might raise the elevation sufficiently to cause the overtopping of the levees. The possibility of such action should justify a thorough study of this point before a final decision on retarding basins is made. The existing bed is the product of the filling and scouring of the river with its present sediment load, and any conditions, such as the construction of the retarding basin, which would add materially at frequent intervals to the sediment load, would probably cause an increase in the average level of the river-bed. The raising and lowering of the bed levels mentioned in the paper are believed to be a demonstration of the sensitiveness of the river to changes in the character or amount of the sediment load.

[28b] *Proceedings*, Am. Soc. C. E., March, 1939, Fig. 36, p. 553.

[29] *Journal*, Assoc. of Chinese and American Engrs., Vol. XVIII, March–April, 1937, p. 130.

The authors have presented a very complete discussion of the Yellow River problem from the standpoint of the water stream, but further consideration might be given to the problem from the standpoint of the sediment stream. Viewing the situation broadly, it seems that the sediment load of the Yellow River must be handled in one or more of the following three ways: (1) It may be reduced by keeping the soil in place on the land by erosion-control methods; (2) it can be carried out to sea and deposited on the sea bottom; and (3) it may be stored on the land surface by taking it out of the river. Probably the best solution will be a combination of the three methods.

In a paper entitled "Soil Erosion and River Regulation with Special Reference to the Yellow River,"[30] Mr. Eliassen has discussed at some length the possibilities of reducing the sediment load of the Yellow River by soil-conservation methods. Based on the cheap labor conditions existing in China and the knowledge gained in soil-control engineering in the United States, it seems probable that at least part of the sediment load should be taken care of in this manner. The benefits would be not only that of flood control but also of increased food supply which could probably be produced by the vegetal cover developed in the erosion-control process. While erosion control has no doubt been practiced in the past as a means of conserving agricultural land, its value as a flood-control measure has not been considered, and if financial support were given to the work because of the flood-control benefit it is possible that an extensive program along this line might be feasible. If practically all the sediment could be retained on the land, the clear water in the river would pick up a large volume of sediment from its bed and carry it to the sea, thus enlarging the river channel, and eventually solving the flood problem. It is improbable that anything approaching complete erosion control will be practicable, but something between complete control and the present conditions will probably be a part of the best plan for flood control.

The second method, by carrying the sediment to the sea, will no doubt be a part of any flood-control plan. As long as the Yellow River flows over a bed of fine material it will not be feasible to prevent it from carrying a large load out to sea. Although this disposition does bring in some serious problems, as the authors have pointed out, these difficulties are probably less than those of the third method, and, therefore, deposition in the sea should be encouraged as far as possible. Since the fall of the river in crossing the plain cannot be materially increased without raising the level at the western side of the plain much higher that it is now, it is probable that, aside from maintaining the levees intact and possibly regulating the channel across the plain to keep it deep and straight, little can be done to increase the ability of the river to transport sediment.

If this is the case, the amount that can be carried out to sea under the most favorable conditions will be considerably less than that now brought into the plains section by the river, and unless a major reduction in volume can be made by soil-conservation methods, any long-time plan for Yellow River flood control must include provision for controlled sediment storage. In other words, for complete control of the floods, if more sediment enters the plains section of the

[30] *Journal,* Assoc. of Chinese and American Engrs., January–February, 1936.

river than can be carried out to sea, the excess must be stored by some controlled method. The excess which now exists may be reduced by soil conservation and possibly by producing some increase in the carrying capacity of the river, but unless the excess is entirely eliminated by these methods, disasters cannot be permanently prevented except by means of sediment storage under controlled conditions.

There can be little doubt that in the past the flow of sediment in the river at the western side of the plain has materially exceeded that which passed out of the mouth at the east side, and therefore that there were large quantities of material deposited between these two points. When the river changed its course in 1851, the meager accounts available indicate that the new course to the sea was along the line of a stream which had a much flatter gradient than the Yellow River and therefore flowed at a lower level. The flow through the breach in the dikes entered this low-level stream, and the bed of the Yellow River above the break was cut back, lowering the bed materially. Since this change of the river's course took place, the bed of the new portion of the channel filled considerably, and, as stated by the authors, an adjustment is still taking place in the vicinity of the break. In this process tremendous quantities of sediment have been deposited, which accounts for part of the excess of sediment inflow over outflow in the plains section, as previously mentioned.

Another part of this excess has been deposited on the land by the flow through the breaks in the dikes. When such breaks occur, the heavily silt-laden water flows over the land and deposits tremendous quantities of sediment. In some cases the flow returns again to the Yellow River in a comparatively clear condition and goes on out to sea. In other cases it goes overland to some other stream leading to the sea, but in either event a large part of its sediment is deposited on the plain.

Unless the inflow of sediment at the western side of the plain can be made to equal that flowing into the sea at the eastern side, a levee system alone will not be a permanent solution for the Yellow River floods, since the sediment storage which takes place will raise the bed of the stream and cause progressively higher flood levels until so great a height is reached that levee construction becomes very expensive and insecure. In this condition, if a break occurs, the elevation of the river-bed will be so much higher than the land that the water cannot be forced back into the channel. The raising of the bed will increase slightly the amount of sediment which can be carried out to sea, but since a 5% increase in slope would raise the river level about 15 ft at the western side of the plain, no great increase in average slope can be attained without raising the river at that point to unsafe levels.

Since it seems unlikely that in the plains section of the river the combination of reduction in sediment inflow by soil conservation and the increase in sediment outflow can reduce the excess of inflow over outflow to zero, some form of controlled sediment storage must form a part of any permanent plan for the control of the Yellow River floods. If this is the case, such storage will have to be carried on continuously and sooner or later the basins for the deposit of the sediment will be filled. It appears, therefore, that no set of works can in themselves

be a complete solution of the flood-control problem, but sooner or later other works will have to be constructed and the control of the Yellow River will therefore be a continuing process of construction work.

Another reason why continuous work will be necessary is that as the sediment continues to be carried out to sea, the mouth of the river will move outward, as explained by the authors. The river levees, therefore, will have to be extended seaward and those now near the sea will have to be raised. Although this process will no doubt be slow, it seems to be inevitable. In considering the best plan for controlling Yellow River floods, therefore, it should be realized that no solution will entirely eliminate the necessity of future construction, and therefore works which are obviously not a permanent solution should be considered along with those which may appear to be, but in reality are not, permanent solutions. When one considers the necessity of controlled sediment storage in any plan for permanent flood control, with the consequent expense and difficulties due to opposition of land owners, the advantages of soil conservation from the flood-control standpoint become apparent. Of course, such measures also would be a continual expense but it seems certain that a large program of work to keep much of the soil in its place will be cheaper and more satisfactory than artificially storing the soil after it has reached the river.

The foregoing statements must not be construed as showing that better levees should not be built along the Yellow River or that those now in existence should not be maintained. The damage caused by a levee break is so great that better levees might pay for themselves long before they became inadequate. It may not be possible for some years in the future to finance the works necessary for complete control. What the writer wishes to emphasize is that eventually sediment storage will probably be necessary, and that the best possible plan of action is more likely to be found both for the immediate future and for more distant times if all the factors are considered. Therefore, every effort should be made to collect the information necessary for a complete picture of the problem. In the writer's opinion this would include the collection of more data on the composition and concentration of the sediment carried by the river and its tributaries and a better working out of the laws governing sediment transportation.

Sediment storage for the improvement of streams is no new thing in China. On the Yung Ting Ho above Tientsin there is a large area in which silt has been depositing for many years and which has permitted the Hai Ho from Tientsin to the sea to maintain a navigable channel. The writer has not been able to find any statements as to when this sediment-storage basin was constructed and whether or not it was built with that function in view, but in any event it has served very well for this purpose until recent years when it has become so full that it is no longer effective and the channel of the Hai River has been badly deteriorated. To replace this sediment-storage basin a new one has been constructed and, it is understood, is now functioning satisfactorily.

Three principal plans for controlled storage of sediment might be developed: (*a*) Storage in reservoirs west of the plain, (*b*) storage in basins along the river in the plain, and (*c*) storage at the west side of the plain. The authors have

mentioned good sites for reservoirs in the section between the Peiping-Hankow Railway and the mouth of Wei Ho, and also in the vicinity of the Hu Kou Falls. Sediment storage could probably be obtained at reasonable cost at these sites. As stated before, the reservoirs would undoubtedly fill rapidly and eventually new storage space would be required. In the early stages of filling they would serve admirably for retarding basins and would have a dual purpose—the reservoirs could be used for the generation of water power and in the early stages of filling would have available considerable storage and pondage space. As they became filled, however, the power generating plants would have to be operated on a run-of-river basis. Thus the reservoirs might be used either for combined retarding basin and silt storage, with the retarding ability eventually being lost, or as combined water power and sediment storage, with part of the water power benefits eventually being lost. Since the vicinity of these basins is well supplied with coal, it would seem that the combination of sediment storage with retarding basin action would probably be the best. The use of these basins as detention basins would also permit their use for flushing out the river down stream by storing clear water during low-flow times and releasing it with higher flows during short periods.

The necessary sediment storage could be accomplished by deposit in predetermined areas in the plain section during floods, when the water level was high enough to permit overflow of the land along the stream. By building a pair of dikes parallel to and outside of the present ones (where secondary dikes are not already in existence), with cross-dikes connecting the outside and inside dikes at intervals, a series of basins could be formed in which the water could be let in at the upper end through sluice-ways and the desilted water let out at the lower end. Such a scheme would build up the land along the river banks and thus add to the security from disastrous levee breaks. The desilted water released at the lower end of these basins, upon flowing back to the river, would pick up a load of sediment from the river banks and bed, and thus enlarge the capacity of the river channel. The amount of material picked up, however, would probably be materially less than that deposited, since the former material would average much coarser than the latter. The construction of the second dike system would be a considerable expense, although it would not have to be as heavy as the primary system, since in case of a break only the water in the desilting basin would be released. The principal disadvantage of this form of sediment storage is the damage resulting from taking so large an area of land out of cultivation or the periodical destruction of the crops which would occur if this land was cultivated, since the Yellow River floods come during the growing season. This plan could be put into operation only under a strong central government and would no doubt meet with a great deal of opposition from the present owners of the land included in these storage basins.

A third plan for storage of the surplus sediment would be to construct basins where the river emerges from the hills, causing the river to form a sort of debris cone by spreading it out over the plain in this vicinity under controlled conditions. This plan would háve the same disadvantages as that previously de-

scribed but might be cheaper. This form of sediment storage might be combined with retarding basins, the sediment carried out from the retarding basins after the floods being desposited under controlled conditions on the plain. All three plans have the disadvantage that they would require continuous work, but no form of flood protection for the Yellow River seems feasible which can be constructed immediately in its final form.

The authors have presented a large amount of data on the silt load of the Yellow River, but no data have been given on the size of the particles of which this silt was composed. The writer understands that analyses of numerous silt samples have been made by one of the authors, but that the data are inaccessible because of the war. It would be a great misfortune if this information should

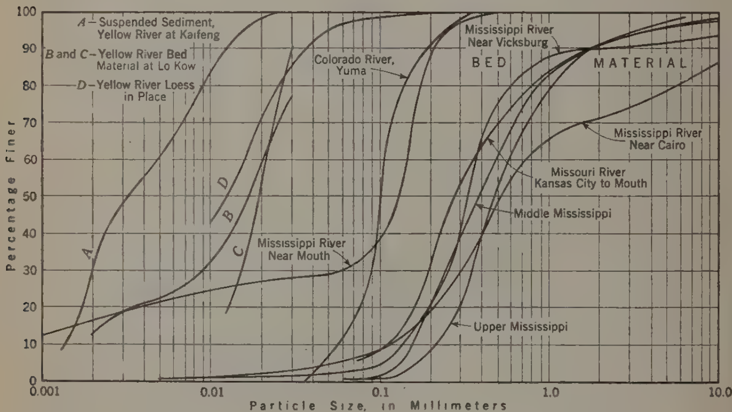

FIG. 37.—COMPARISON OF SEDIMENTS, YELLOW AND AMERICAN RIVERS

be permanently lost, since sedimentation investigations are showing conclusively that a knowledge of the particle size or settling rate is essential in studying sedimentation problems.

Very little information on the size of sediment in the Yellow River is available, but three curves showing the compositions of Yellow River sediment are shown in Fig. 37, the data being taken from "Die Rigelung des Hwangho," by Li Fu Tu. The left-hand curve, for material at Kaifeng, is probably typical of suspended material in the river. The remaining curves give analyses of bed material of a number of the rivers in the United States. The finest of these is the bed material of the Colorado River near Yuma, Ariz.

This diagram strikingly illustrates the difference in the character of the Yellow River sediment and that found in the rivers of this country. The median diameter of these various samples is as follows:

Sample	Median diameter, in millimeters
Yellow River suspended sediment at Kaifeng	0.0033
Yellow River loess in place	0.012
Yellow River bed material at Lo Kou Chen (average)	0.018
Colorado River bed material	0.10
Missouri River bed material	0.28
Lower Mississippi River bed material at mouth	0.13
Lower Mississippi River bed material at Vicksburg, Miss.	0.33
Lower Mississippi River bed material near Cairo, Ill.	0.53
Middle Mississippi River bed material	0.38
Upper Mississippi River bed material	0.48

A comparison of bed material shows that the average of a sample from the Yellow River was about one-fifth the size of the Colorado River bed material, one-seventh that of the Mississippi at its mouth, and from one-sixteenth to one-twenty-seventh that of the various other rivers for which data were given. It seems obvious that the reason why the Yellow River can carry such heavy loads of sediment is that the material carried is so extremely fine. These samples were taken in the plain section of the river, but the gorge section above Mengtsin carries a certain amount of quite coarse material, as shown by the fact that this was sufficiently large to break the glass bottles used in the sampling device, and metal bottles had to be substituted. This coarse material apparently settles out in the vicinity of Mengtsin in a sort of debris cone and does not reach the plains section of the river.

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DISCUSSIONS

GRAPHICAL ARCH ANALYSIS APPLICABLE TO ARCH DAMS

Discussion

BY I. M. NELIDOV, ASSOC. M. AM. SOC. C. E.

I. M. NELIDOV,[9] Assoc. M. Am. Soc. C. E. (by letter).[9a]—The method of graphical analysis given in this paper represents a development over the similar methods in existence, in that it offers, in one complete diagram, the graphical determination not only of forces but also of deformations.

In so far as they refer to Equations (1) to (13) the formulas and coefficients in Table 1 are correct.[9b] If a more simple case of a circular arch of constant cross-section under uniform load is taken, the same expression for crown thrust is obtained as is given by the late William Cain,[10] M. Am. Soc. C. E. The derivations, being somewhat voluminous, are not given herein. In reference to the finite voussoirs the writer wishes to emphasize that the selection of the center of the voussoir, at one time for one purpose, and of the ends of the voussoir, at another time for another purpose, does not help to mechanize the process, which in other respects the authors succeeded in doing. The primary cause of this selection is the small number of the voussoirs and the desire to maintain accuracy. It might be better to double the number of voussoirs and thus avoid the complicated assumptions of centers and sides.

It is evident that the authors were greatly interested in evolving the graphical multiplication and integration. The subject of graphical operations of mathematical functions has attracted investigators in the past.[11] Various operations can be performed, such as addition and subtraction, multiplication and division, raising to a given power, evaluation of radicals, operations with trigonometric functions, integration, etc. It is commendable that the authors

NOTE.—This paper by Carl H. Heilbron, Jr., Assoc. M. Am. Soc. C. E., and William H. Saylor, Jun. Am. Soc. C. E., was published in January, 1939, *Proceedings*. This discussion is published in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[9] Senior Engr. of Hydr. Structure Design, State Div. of Water Resources, Sacramento, Calif.

[9a] Received by the Secretary April 19, 1939.

[9b] Correction for *Transactions:* In Equations (13) change "H_t" to "H_c."

[10] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 522.

[11] "Die Graphische Methode," by C. Runge, Berlin, Germany; also, "Manual for Graphical Computations," by V. P. Pharmakovsky (in Russian).

offered a complete explanation of the graphical processes involved in this method.

As to the "trial force," the writer wishes to remark that this is partly a misnomer and that it is not presented with sufficient clarity. It has been known in the theory of arches[12] that all the forces in an elastic arch can be divided into two classes, the first resulting from the rigid arch and the second resulting from its elastic action. The forces then are called "main" or "original," and "secondary." Using the notation of the paper with the subscript t to indicate the original forces, the expressions for any section of the arch are:

$$P = P_t + P_0 \qquad (18a)$$

$$S = S_t + S_0 \qquad (18b)$$

and

$$M = M_t' + M_0 \qquad (18c)$$

in which the subscript 0 refers to the forces caused by the elastic action, and, M_t' is accented in order to distinguish it from M_t as shown subsequently in Equation (21).

For the crown section Equations (18) become:

$$H = H_t + H_0 \qquad (19a)$$

$$V = V_t + V_0 \qquad (19b)$$

and

$$M_c = M_t' + M_0 \qquad (19c)$$

In Equations (18) and (19) P_t, H_t, S_t, V_t, and M_t' are known, being the forces in the rigid arch resulting from the external forces. Thus:

$$P_t = P_e + H_t \cos \alpha \qquad (20a)$$

$$S_t = S_e + H_t \sin \alpha \qquad (20b)$$

and

$$M_t' = M_e + H_t\, y + V_t\, x \qquad (20c)$$

However, the authors defined M_t as:

$$M_t = M_e + H_t\, y + V\, x \qquad (21)$$

thus including, in V, both V_t and V_0. For illustration, in applying Equations (18) to (21) to the case of a circular arch with neutral radius r, constant thickness t, half of the central angle α_1, and uniform radial load p:

$$H_t = \frac{p\, 2\, r^2 \sin^2 (0.5\, \alpha_1)}{r\, (1 - \cos (0.5\, \alpha_1))} = p\, r \qquad (22a)$$

$$V_t = 0 \qquad (22b)$$

$$P_t = 2\, p\, r \sin^2 (0.5\, \alpha) + p\, r \cos \alpha = p\, r \qquad (22c)$$

$$S_t = 2\, p\, r \sin (0.5\, \alpha) \cos (0.5\, \alpha) - p\, r \sin \alpha = 0 \qquad (22d)$$

and,

$$M_t' = - 2\, p\, r^2 \sin^2 (0.5\, \alpha) + p\, r^2 (1 - \cos \alpha) = 0 \qquad (22e)$$

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[12] "Solution of Circular Arch Under Water Load," by Emil Mörsch, Zurich, Switzerland, *Schweizerische Bauzeitung,* Vol. 51, 1908.

In other words, the forces with the subscript t are always known because they are obtained from ordinary statics and no "trials" are needed.

In the case of irregular loading and of the arch, Equations (22) may be obtained graphically from the force diagrams of statics. Referring to the method of obtaining the moment M_t by a summation of the values of shears, as shown in Table 2, Columns (11) and (12), it may be noted that the moments M_t' may be obtained directly by graphics, by constructing a force polygon and an equilibrium polygon for the forces acting and superimposing the latter on the neutral line of the arch.

In conclusion, what interests the writer is the statement by the authors that, to include the effect of the non-uniform temperature, one must add to M_t, due to the external forces, the expression $\frac{dT}{dt}\,\epsilon\,E\,I$. The writer suggests that the authors verify it by applying their formulas in Table 1 to the simple case of a circular arch of neutral radius r, uniform thickness t, half the central angle α_1, and finding a total crown thrust H due to this change of temperature.

The expression for H is as follows:[13]

$$H = \left[\Delta x - \theta\, r \left(1 - \frac{\sin \alpha_1}{\alpha_1}\right)\right] \frac{2\,E\,I}{D\,r^3} \qquad (23)$$

in which the horizontal displacement of the crown section, due to the effect of non-uniform temperature, is

$$\Delta x = r^2\, \epsilon \frac{dT}{dt} (\alpha_1 - \sin \alpha_1) + r\, \epsilon \left(t_i + \frac{t}{2}\frac{dT}{dt}\right) \sin \alpha_1 \qquad (24a)$$

The rotation of the crown due to the same effect is

$$\theta = \frac{dT}{dt}\, r\, \epsilon\, \alpha_1 \qquad (24b)$$

Coefficient D is

$$D = \left(1 + \frac{I}{t\,r^2}\right)\left(\alpha_1 + \frac{\sin 2\,\alpha_1}{2}\right) - \frac{2 \sin^2 \alpha_1}{\alpha_1} + n \frac{I}{t\,r^2}\left(\alpha_1 - \frac{\sin 2\,\alpha_1}{2}\right) \qquad (24c)$$

and, t_i is the temperature at the intrados, in degrees Fahrenheit.

[13] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), pp. 492 and 558.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

SIMPLIFIED WIND-STRESS ANALYSIS OF TALL BUILDINGS

Discussion

BY CHARLES B. WINICK, ASSOC. M. AM. SOC. C. E.

CHARLES B. WINICK,[16] ASSOC. M. AM. SOC. C. E. (by letter).[16a]—An able and simplified method of computing wind stresses is presented in this paper. Designers of buildings have long been in search of a simple method of obtaining shears, reactions, and bending moments due to wind loads for the various members of a building frame. Since the development of the Hardy Cross method[17] of moment distribution, extensive literature has been written on the subject. The application of the Cross method to obtain shear, reaction, and moment distribution due to wind loads on a building frame is relatively simple, especially if it is not extended to a high degree of accuracy. Numerous attempts have been made, more or less successfully, to obtain "accurate analyses" by some one or other "exact method." The writer is cognizant of the fact that the problem of stress distribution due to wind loads, even in a relatively simple building, does not lend itself readily to an exact solution. Flooring, partitions, stair openings, elevator shafts, concrete slabs, and numerous other factors invariably affect the stiffness of the structure. It is questionable, therefore, whether the time consumed in arriving at a so-called accurate solution, through the application of some "exact method," is justifiable and consistent with the accuracy required in a problem of such uncertain nature. The method as outlined by Mr. Gottschalk is sufficiently accurate to serve the purposes of almost any type of building when the moments of inertia of the several members involved are constant. A designer familiar with building structures can readily apply this method, and it is questionable whether results obtained by any other complicated "exact method" are superior to it.

The writer has undertaken to recompute the values given by the author, as shown in Figs. 2 and 3, with the aim of applying some "short cuts" wherever

NOTE.—This paper by Otto Gottschalk, Esq., was published in December, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1939, by Samuel T. Carpenter, Jun. Am. Soc. C. E.

[16] Engr., Queens Midtown Tunnel, PWA, New York, N. Y.

[16a] Received by the Secretary April 12, 1939.

[17] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Vol. 96 (1932), p. 1.

TABLE 1.—RELATIVE RESISTANCE VALUES OF BUILDING COLUMNS TO UNIT LATERAL DISPLACEMENT, USING UPPER FLOOR BEAMS ONLY.

Nomenclature	Columns Between Floors $(n+1)$ and (n)				Beam AB; $L_1=15$ ft		Beam BC; $L_2=20$ ft		Beam CD; $L_3=25$ ft	
	Column A	Column B	Column C	Column D	at A	at B	at B	at C	at C	at D
$k=\frac{I}{h}$ or $k=\frac{I}{l}$	2.5	2.2	4.6	3.3	3.2		3.7		4.7	
$\frac{k_B}{k_B+k}$ Upper beams equal to $\theta_A-\theta_B$	....	....	....	....	$\frac{3.2}{3.2+2.5}=0.562$	$\frac{3.2}{3.2+2.2}=0.592$	$\frac{3.7}{3.7+2.2}=0.627$	$\frac{3.7}{3.7+4.6}=0.446$	$\frac{4.7}{4.7+4.6}=0.505$	$\frac{4.7}{4.7+3.3}=0.588$
$k(\theta_A-\theta_B)$	....	....	....	....	$0.562\times2.5=1.405$	$0.592\times2.2=1.302$	$0.627\times2.2=1.379$	$0.446\times4.6=2.051$	$0.505\times4.6=2.323$	$0.588\times3.3=1.940$ Total $=10.40$
Relative resistance values	....	....	....	....	$\frac{1.405}{10.4}=0.135$	$\frac{1.302}{10.4}=0.125$	$\frac{1.379}{10.4}=0.133$	$\frac{2.051}{10.4}=0.197$	$\frac{2.323}{10.4}=0.224$	$\frac{1.940}{10.4}=0.186$ Total $=1.00$
Relative resistance of columns between (n) and $(n+1)$ stories to lateral load $P_{(n+1)}=1$	0.135	$0.125+0.133=0.258$	$0.197+0.224=0.421$	0.186 Total $=1.00$	....	....	....	....	....	....
Horizontal shear T between (n) and $(n+1)$ floors in kips	$28.8\times0.135=3.9$	$28.8\times0.258=7.4$	$28.8\times0.421=12.1$	$28.8\times0.186=5.4$ Total $=28.8$	....	....	....	....	....	....
Bending moments M at top in kip-feet; inflection points at mid-heights	$3.9\times7=27.3$	$7.4\times7=51.8$	$12.1\times7=84.7$	$5.4\times7=37.8$	....	....	....	....	....	....
Total bay resistance	....	....	....	....	$0.135+0.125=0.260$		$0.133+0.197=0.330$		$0.224+0.186=0.410$ Total $=1.00$	
Overturning wind moment $28.8\times7=201.6$ kip-ft. Therefore, shear of bays in kips	....	....	....	....	$\frac{201.6\times0.260}{15}=3.5$		$\frac{201.6\times0.33}{20}=3.3$		$\frac{201.6\times0.410}{25}=3.3$	
Reaction R in kips	-3.5	$3.5-3.3=0.2$	$3.3-3.3=0$	3.3	....		....		....	

possible, and has found some interesting results. For this purpose, all computations are referred to Fig. 2. The writer has found it difficult to refer to Fig. 3 because it is not consistent with Fig. 2. Thus, the $(n + 1)$th story of Fig. 3 corresponds to the nth story of Fig. 2, etc. Reference to both Figs. 2 and 3 leads to considerable confusion and, as a substitute, the writer developed separate tables for each analysis of each floor shown in Fig. 2. All values obtained and their methods of computation were arranged as shown, for example, in Table 1. The complete computations were made in logical order, with painstaking care. Before discussing some of the results it is well to emphasize the fact that the method is only approximate but sufficiently accurate for all practical purposes.

As the author states, when $S_B = \infty$, $m = 0$ and $S = k$; and, when $S_B = k$, $m = 0.25$ and $S = 0.875$. The range of values of m, therefore, is between 0 and 0.25; that of $\frac{S}{k}$ is between 0.875 and 1.0; and, therefore, the assumptions that—(a) The conditions of rigidity at the tops of columns are sufficiently similar to those at the bottom for a typical story, and (b) that, in all structural frames, for lateral movements of floors, $m = 0$ and $S = k$—will produce a small error in the final computation of the column shears, moments, and reactions.

As shown in Fig. 3, the relative resistances of building columns of a typical floor, to unit lateral displacement, were computed by utilizing both the upper and lower floor beams which the column spans. The writer computed similar values by using either the upper or lower floor beams and also the beams of both floors, with remarkable agreement in results.

Table 1 shows the relative resistance values of building columns for the $(n + 1)$th story to a lateral load of $P_{n+1} = 1$ at the top as shown in Fig. 2 using upper floor beams only. The first line shows the k-values equal to $\frac{I}{h}$ (or $\frac{I}{l}$, as the case may be). The second line gives the $(\theta_A - \theta_B)$-values at the various points. Thus, starting with Point A, $\frac{k_B}{k_B + k} = \frac{3.2}{3.2 + 2.5} = 0.562$; $\frac{3.2}{5.4} = 0.592$; $\frac{3.7}{5.9} = 0.627$, etc. Next, the $k(\theta_A - \theta_B)$-values are given which are equal to $0.562 \times 2.5 = 1.405$; $0.592 \times 2.2 = 1.302$; $0.627 \times 2.2 = 1.379$ etc., the total being equal to 10.40. The relative resistances were computed as shown on the next line in the table. Then the relative column resistances to the lateral load $P_{n+1} = 1$ were computed; and, in a similar manner, these values were computed using the lower floor beams, the results being 0.135; $0.126 + 0.132 = 0.258$; $0.199 + 0.223 = 0.422$; and, 0.185. Finally, using both floors as the author does, the corresponding values were: 0.135; $0.126 + 0.132 = 0.258$; $0.198 + 0.223 = 0.421$; and, 0.186. Table 2 (*a*) shows the comparison of these relative resistance values as obtained for each case and further compares them with the same values as obtained by the author. The disagreement between the writer's and author's values is probably due to the fact that Mr. Gottschalk has used the slide-rule. In order to make certain and to ascertain the probable percentage of error by comparing each case, all computations were extended at least three decimal places and in some instances even five places.

TABLE 2.—COMPARISON OF RESISTANCE VALUES OF BUILDING COLUMNS

Column	Case I, using the upper floor beams (1)	Case II, using the lower floor beams (2)	Case III, using both floors (3)	Gottschalk analysis (4)
(a) FOR A LATERAL LOAD $P_{n+1} = 1$				
A	0.135	0.135	0.135	0.135
B	0.125 + 0.133 = 0.258	0.126 + 0.132 = 0.258	0.126 + 0.132 = 0.258	0.125 + 0.140 = 0.265
C	0.197 + 0.224 = 0.421	0.199 + 0.223 = 0.422	0.198 + 0.223 = 0.421	0.198 + 0.217 = 0.415
D	0.186	0.185	0.186	0.185
Total	1.000	1.000	1.000	1.000
(b) FOR A LATERAL LOAD $P_n = 1$				
A	0.121	0.119	0.120	0.121
B	0.126 + 0.133 = 0.259	0.124 + 0.114 = 0.238	0.125 + 0.122 = 0.247	0.248
C	0.197 + 0.226 = 0.423	0.183 + 0.254 = 0.437	0.189 + 0.242 = 0.431	0.432
D	0.197	0.206	0.202	0.203
Total	1.00	1.00	1.00	1.00

Table 2 (*a*) shows a close agreement in the first three columns, which would lead to the definite conclusion that the relative resistance values of columns may be obtained by taking into account either the upper floor beams, the lower floor beams, or the beams of both floors. It is interesting to compare the author's results with those of the writer as shown in Columns (3) and (4) of Table 2 (*a*) (since both have been developed under the same assumptions) in order to ascertain the probable error caused by the use of the slide-rule. Thus, at Column B $\frac{0.265}{0.258} = 1.027$; and, at $\frac{415}{421} = 98.6$; or, 2.7% at Column A and 1.4% at Column B. In other words, this comparison shows that the slide-rule may cause an error of as much as 3 per cent.

Again referring to Table 1, the horizontal shears, T, in kips, were computed; and, since they are a direct function of the relative column resistances which are the same in all three cases, the results are: 3.9, 7.4, 12.1, and 5.4; and, if bending moments are computed with inflection points assumed at mid-height of columns, the corresponding values (in kip-feet) are: 27.3, 51.8, 84.7, and 37.8. In order to obtain the vertical reactions, the shears at the bays were computed as shown on the next to last line in Table 1. The last line, completing the table, gives the vertical reactions at the four columns of the structure.

In a similar manner, the writer computed the relative resistance values of building columns for the nth story, due to the lateral load $P_n = 1$ at the top, as shown in Fig. 2. These values were computed, first taking into account the upper floor beams only, as in Table 1. The same values utilizing the lower floor beams, and the beams of both upper and lower floors, were derived as in Table 1.

Table 2 (*b*) affords a comparison of the relative resistance values of columns as obtained in each case and also compares it with the values obtained by the author.

A comparison of values in Table 2 (*b*) (see Table 3) shows a maximum error of only 4.86% in the extreme case, which would again lead to the conclusion

TABLE 3.—COMPARISON OF PERCENTAGE ERRORS IN RELATIVE RESISTANCE VALUES

For column:	Cases I and II, Respectively, with Case III, Yield:			
	Case I		Case II	
	Computation	Percentage error	Computation	Percentage error
B	$\frac{12}{247} \times 100$	+ 4.86	$\frac{9}{247} \times 100$	− 3.65
C	$\frac{8}{431} \times 100$	− 1.86	$\frac{6}{431} \times 100$	+ 1.39
D	$\frac{5}{202} \times 100$	− 2.48	$\frac{4}{202} \times 100$	+ 1.98

that the relative resistance values of columns may be obtained by taking into account either the upper floor beams or the lower floor beams or the beams of both floors.

Table 4 gives a comparison of the horizontal shears T, the moments M, and the vertical reactions R, for all columns of the structural bay, for each case,

TABLE 4.—COMPARISON OF HORIZONTAL SHEARS, BENDING MOMENTS AND VERTICAL REACTIONS

Column	(*a*) Horizontal Shear, T						(*b*) Bending Moment, M						(*c*) Vertical Reactions, R			
	Case I		Case II		Case III; kips	Gottschalk; percentage error	Case I		Case II		Case III; kip-feet	Gottschalk; kip-feet	Case I; kips	Case II; kips	Case III; kips	Gottschalk; kips
	Kips	Percentage error	Kips	Percentage error			Kip-feet	Percentage error	Kip-feet	Percentage error						
A	3.9	0.0	3.9	0.0	3.9	3.9	31.2	0.0	31.2	0.0	31.2	31.2	−7.6	−7.5	−7.5	−7.7
B	8.4	+5.0	7.7	−3.8	8.0	8.1	67.2	+5.0	61.6	−3.8	64.0	64.8	+0.0	+0.7	+0.3	+0.6
C	13.7	−2.1	14.1	+0.7	14.0	13.9	109.6	−2.1	112.8	+0.7	112.0	111.2	−0.2	−1.7	−1.0	−1.0
D	6.4	−0.2	6.7	+0.3	6.5	6.5	51.2	−0.2	53.6	+0.3	52.0	52.0	+7.8	+8.5	+8.2	+8.1

with the probable percentages of error. The maximum probable error in computations of bending moments and shears is only 5% and is sufficiently accurate for all practical purposes. The percentages for vertical reactions were neglected since the reactions are so small that an error of even 100% would not affect the design materially.

An attempt was also made to obtain similar values for the three cases for the ground floor, but since the columns at this point are assumed to be entirely fixed, it can readily be seen that in order to obtain the relative resistances, both ends of columns must be considered. However, the computations for the ground floor were tabulated as in the other cases, taking into consideration:

(1) The upper beams only, (2) the lower end of columns only, and (3) both ends as solved by the author. The results (not included herein) showed sufficient disagreement to emphasize the necessity of ascertaining the condition of column fixity at both ends before analysis is made.

The beam shear method of obtaining the approximate shear, T, in beams and reactions, R, in columns, as outlined by Mr. Gottschalk, is rather ingenious. This method is relatively simple, and with little experience an analysis can be made in a relatively short time. The writer has checked the computations, extending them to a greater degree of accuracy and finds no serious discrepancies.[17a]

In conclusion, it may be added that the very simplicity and brevity of the theoretical analysis and derivation of formulas, as well as the ready applicability of the method to practical problems, are indeed a worthy contribution to such a highly controversial subject, and the author is to be congratulated for his effort to lighten the burden of engineers and designers engaged in this field of endeavor. Undoubtedly many short cuts will be found as familiarity with either the "column-shear" or "beam-shear" method is gained. Thus, since nearly the same relative resistance of the columns to unit lateral displacement was obtained for a given set of columns, in a particular bay, by utilizing the beams of the upper, lower, or both floors, it can be seen readily that intelligent application of the method will lead the experienced engineer to choose that set of k-values in the beam system which will result in simple computations, thereby saving considerable time and minimizing the chances of errors.

[17a] Corrections for *Transactions:* In Fig. 4, Line 12, change "4.43" to "44.3"; and in Fig. 4, Line 15, change "− 76.6" to "− 70.6."

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DISCUSSIONS

THE RISK OF THE UNEXPECTED IN SUB-SURFACE CONSTRUCTION CONTRACTS

Discussion

BY MESSRS. LAZARUS WHITE, AND T. KENNARD THOMSON

LAZARUS WHITE,[13] M. AM. SOC. C. E. (by letter).[13a]—The paper by Mr. Herwitz is very timely, of great value to the engineer, and is deserving of a full discussion. In the writer's experience very little on this subject is taught in engineering schools; yet contracts and specifications are primarily created by the engineer through his investigations, designs, and specifications, although the contract part of the documents may be the work of lawyers—and here "comes the rub." The engineer responsible for a sub-surface contract is often too hampered by a lack of time or funds to investigate sub-surface conditions properly; he then draws up designs, as nearly as possible fitting the conditions as known or assumed by him. He feels that if conditions are not actually encountered as assumed, claims for extra compensation will be filed by the contractor; and he relies upon the lawyer to protect the owner from such claims —hence, the "exonerating clauses" or "alibi clauses" found in many contracts. Thus, to the business risks and labor risks, which a contractor must assume, are added risks which do not properly belong to him. The owner should provide sufficient time and means for the engineer to ascertain the underground conditions properly; or, if this is impracticable, the owner should assume the extra cost of performing the work under conditions different from those upon which the contract was based. The engineer degrades his profession by hiding behind tricky legal defenses so that extra expenses (which, in fairness, the owner should assume) are thrown upon the contractor whose bid prices are usually narrowly competitive.

The engineer is properly the agent of the owner, and it is perfectly proper that as the owner should profit by the skill and economies of design of the engineer he should also assume the extra costs when the designs do not fit the assumed conditions, and when the conditions are different.

NOTE.—This paper by Oren Clive Herwitz, Esq., was published in January, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1939, by Messrs. Alonzo J. Hammond, Frederick W. Newton, David A. Molitor, F. B. Marsh, and Evan S. Martin; and May, 1939, by Messrs. C. Maxwell Stanley, and Francis J. Morgan and Frederick C. Zeigler.

[13] Pres., Spencer, White & Prentis, Inc., New York, N. Y.

[13a] Received by the Secretary April 14, 1939.

The "Synopsis" states that in the early days it was customary for the contract to provide that the risk of the unexpected should fall on the contractor. Broadly worded contract clauses took the place of scientific and careful investigation. Unfortunately this practice has persisted so that it is still common in contracts originating in certain organizations, as for instance, in the following note, attached to the principal sheet of an important contract.

"This record of borings is issued for the convenience of contractors. It is not a contract document. No figures nor notes are guaranteed. Sections and profiles are interpolated and are not exact."

Although Mr. Herwitz cites practically all the cases bearing upon sub-surface contracts and clearly defines the meaning of the decisions, there is no clear thread running through these cases and as he states in the last analysis "each case must rest upon its own distinctive facts."

The writer, from personal knowledge of several of the cases cited and with forty years experience in such sub-surface contracts, has sought to find some common logic in the series of apparently contradictory decisions, and has come to this conclusion that: (1) When the explorations, engineering and design, and the contract and specifications have been well done, the owner seldom loses; and (2) good and thorough engineering is the best protection and in most cases discourages litigation. Such contracts are consistent with the old definition of a contract being a "meeting of the minds." In other words a proper contract is that in which both parties agree upon a set of conditions upon which a price is based. Such contracts are not so uncommon; specifications of the U. S. Engineer Department contain clauses enabling adjustments and extra compensation to be made when "changed conditions" are encountered, or when designs must be altered because of "changed conditions."

Mr. Herwitz states that "reports of borings and similar data may return to plague both owner and contractor, even when given in the utmost good faith and with careful attention to the best modern practice." This seems to be apparent from the cases cited but again, in the writer's experience, the cases lost by the owners were not supported by well-prepared or well-explored contracts. The main difficulty with underground investigations is not that they are improperly made as much as improperly interpreted and classified—classifications made by a foreman rather than by a trained geologist. The Catskill Aqueduct crossed many buried gorges and difficult terrain; yet it was so well explored, and its geology so well investigated that little of the unexpected was encountered, and the few suits started were lost in the Courts. Of course the writer does not maintain that there is no residue of uncertainty in sub-surface work; but he believes that an arbitration clause is desirable to adjust claims arising from such uncertainties. A board of experts can come quickly to a just decision as compared to an ordinary jury. Almost manifest injustices will be avoided, as where public construction organizations use the sovereign power of the State to bar just claims or where clever lawyers write in trick clauses which serve as traps for the unwary.

In conclusion the writer would emphasize that the engineer prepare his contract thoroughly, act as a just judge in interpreting it, and forget that his judgment is "final and conclusive."

T. KENNARD THOMSON,[14] M. AM. SOC. C. E. (by letter).[14a]—Mr. Herwitz's very interesting paper draws attention to many cases in which Courts give diametrically different judgments on what, to the laymen, would appear to be identical cases. Most engineers have all had similar experiences.

The English Law Courts have the reputation (former Chief Justice Lord Reading has so stated within the writer's hearing) of trying to settle law suits on the equity of the case, instead of on technical legalities, as is so often done in the United States.

All that engineers can do is to try to be just to both client and contractor. In fifty-six years of experience the writer can recall only three or four cases in which the engineer deliberately tried to cheat the contractor. Few, if any, other bodies of men can equal or surpass such a record.

An engineer should give the bidders as much information as possible, and should explain, fully, the limitation of such information. It is neither wise nor pleasant to accept a bid when the "successful" bidder does not have a reasonable chance of making an honest profit. If the engineer furnishes the results of test "wash borings" he should explain that such borings give no indication of what kind of material lies under the bottom of such test borings.

In many cases the record states that the boring stopped on "rock or boulder." In some cases, such as Lower Manhattan, if the "wash boring' was made by an experienced engineer, he could tell which it was because he could wash up samples of New York bed-rock but could make no impression on a New York boulder.

With a sample smaller than a cherry he could state whether or not it was New York rock; but, a "core" or diamond-drill boring, penetrating from 3 ft to 5 ft into the rock, would be necessary for him to ascertain whether or not it was bed-rock in its original position, and not a misplaced or deposited piece of rock.

It is frequently difficult to interpret, correctly, the results of borings made by others. The writer once asked a contractor (who had, at that time, probably made more borings in New York than any other man) how much quicksand and hardpan he had found on a certain lot. The contractor replied, "none of either" and then showed the samples which confirmed what the writer already knew—that there was nothing but quicksand and hardpan above the rock at that place.

In some parts of Manhattan, an experienced man would know that he could reach hardpan with "wash borings" and find a fairly uniform level. At the same site the surface of the rock, even a foot or two away, might be entirely different. The writer recalls one case in which there was a drop of 6 ft in a horizontal distance of 2 ft. In such places the expense of many diamond-drill or "core" borings would scarcely be justified. At other points on Manhattan "wash borings" would be worthless, and nothing but "core borings" or test pits should be used.

The writer's practice for caisson work, where he is satisfied with the test borings, etc., is to call for a lump-sum bid to a certain depth, and for a price

[14] Cons. Engr., New York, N. Y.

[14a] Received by the Secretary April 21, 1939.

per cubic yard for work done below that depth. On one job he spent hours convincing the other members of the Board that they should make the estimated cost on the basis of \$25 per cu yd to a stated depth and \$50 per cu yd for all work done below that depth.

At first they could not see why so abrupt an increase should be made until it was explained that \$25 was the average cost from zero to the stated depth, including plant, etc., and that \$50 was a fair price for the greater depth. The irony was that, after convincing the Board and calling for bids on that basis, one contractor took a chance that he would not have to go below that level, and bid a flat \$25 per cu yd. As an experienced contractor, he should have known better because the information was based upon "wash borings" only, which cannot be depended upon, with very rare exceptions, to reach bed-rock.

The call for bids asked for a price, per cubic yard, for 6 000 cu yd, more or less, below the stated depth. If he had bid \$50 per cu yd for that item he would have received the contract anyway. The contractor lost money on that job, and would have lost much more if the Board had compelled him to go deeper as it had the right to do. It happened that the Board did not think it necessary in the interest of safety.

Experienced engineers have occasionally succeeded in carrying "wash borings" to bed-rock, through boulders and other hard material, by using dynamite or powder.

As an example of an unfair specification, a contractor once bid on some excavation work to be done by steam shovel at, say, 20 cents per cu yd. In that case the engineer had the privilege of changing the slope of the excavation if he considered it necessary, even after the excavation had been completed. This meant that the contractor had to flatten the slope by pick and shovel at many times the contract price, and sometimes more than once at the same site. Although the engineer knew this was a rank injustice to the contractor (it was on public work), the engineer had no authority to pay more than the contract price.

Needless to say, books could be filled with descriptions of similar cases.

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DISCUSSIONS

BEACH EROSION STUDIES

Discussion

By Messrs. Elliott J. Dent, and Ralph F. Rhodes

Elliott J. Dent,[7] M. Am. Soc. C. E. (by letter).[7a]—The information that should be secured before undertaking the design of shore-protection works is described fully in this paper. The list of items appears formidable, but the full investigation of one site will ordinarily supply many of the data needed for other sites in the same general vicinity, and the assembly of basic regional data in the files of the Beach Erosion Board often reduces the time, expense, and effort that might otherwise be required when a particular project is under consideration.

The author states that: (*a*) The ratio of length of groin to interval between groins should be between 1 to 1 and 1 to 3; (*b*) spacing closer than 1 to 1 does not injure the beach, but is never economical; and (*c*) spacing greater than 1 to 3 appears to be ineffective. He also states that, subject to the above limits, the correct spacing may be found by drawing through the outer end of a groin a line parallel to the direction of wave approach. The projection of this line upon the beach will give the proper interval.

Along a straight section of beach the axis of wave approach rarely makes an angle of as much as 20° with the perpendicular to the shore. If a line parallel to this axis is drawn through the outer end of a groin, the point of intersection with the beach will be less than half a groin length to leeward of the base of the structure. The use of this rule for the spacing of groins would indicate that, except for economic reasons, the structures should be more than twice as long as the interval between them. This is more than twice as many groins as the author advocates for the closest spacing, and a strict application of the rules would seldom, if ever, permit a spacing of more than 1 to 1.

Fig. 11 shows the profile of a typical groin of the form advocated in 1933 by the Beach Erosion Board for normal use. Experience has since shown that

Note.—This paper by Earl I. Brown, M. Am. Soc. C. E., was presented at the meeting of the Waterways Division at Jacksonville, Fla., on April 21, 1938, and published in January, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1939, by Morris N. Lipp, M. Am. Soc. C. E.; and May, 1939, by Messrs. George A. Soper, and James J. O'Rourke.

[7] Col., U. S. Army (*Retired*), Washington, D. C.

[7a] Received by the Secretary April 19, 1939.

the top of the inner horizontal section may be lowered to at least 1 ft below the normal berm level without affecting the height or width of the finished berm. The sand level shown in Fig. 11 gives conditions before the beach has been built up to the desired form; when the beach has assumed the desired section the groin should be buried from the bulkhead to some point along the sloping part, or along the outer horizontal section. After a set of groins has accomplished the purpose for which it was built a considerable part of each structure will be buried except, possibly, for brief intervals of time following periods of unusually severe erosion. If there is, in fact, a relation between the best length of groin and the best interval between structures, the question arises as to whether the length should be measured from the outer end to the bulkhead, or only to the point where the buried section begins after the beach has assumed the desired form. The latter would seem to be the more logical choice.

It cannot be too strongly emphasized that groins are, in no sense of the word, breakwaters serving to protect the intervening beaches from direct wave attack. However, they may be designed to control the wave-generated currents in contact with the foreshore, and the distance required by diagonal wave attack to create currents of destructive strength is, therefore, an important item in the determination of the proper interval; this distance is vitally affected by exposure, but only slightly so by groin length.

The adequacy of the sand supply is also an important item in the selection of the proper spacing between groins. With a liberal supply of littoral drift the spacing may be much greater than when a scarcity exists.

Each groin inevitably reduces the volume of sand available for the nourishment of the leeward beaches. During the period of accretion along the windward beach the quantity of sand impounded is definitely deducted from the supply of drift. When the impounding capacity of a groin has been exhausted, the quantity of sand passing the site will be unchanged but, unfortunately, part of the sand stream will be detoured around the outer end of the structure in deeper water and at a greater distance from the leeward shore than would be the case on an unobstructed beach. This results in a wastage of sand, and, to minimize this loss, the number of groins should be held to a minimum and the interval between structures should be correspondingly increased. Economy is not the only reason for using the minimum number. The inadequacy of the supply of beach nourishment to the leeward of well-built sets of groins is a phenomenon that has been observed often and has been the subject of much comment.

The proper length of groin for a given location will depend, in part, upon such factors as the width of berm desired, the slope of the foreshore, the tendency for offshore bars to form, and their location. The proper interval between groins will depend largely upon the adequacy of the supply of littoral drift, and the distance required by nature for the formation of destructive wave-generated currents. There appears to be little or no relation between the best length and the best interval.

The art of shore protection has not yet progressed to a state in which past practice can be safely accepted as correct practice, and existing works should be

studied judiciously with a view to the development of better practices. In promoting this principle the Beach Erosion Board has made a photographic and written description of a multitude of existing shore protection structures. A careful examination of this record, including a study of the local conditions, the results accomplished, and the spacing, fails to show the necessity for groins as closely spaced as indicated by the rules presented by the author.

In an address before the American Shore and Beach Preservation Association,[8] the writer advocated the creation of improvement districts, each of which would be responsible for the maintenance of an entire natural subdivision of the shore, usually extending from inlet to inlet. It was assumed that, in most cases, an artificial supply of littoral drift or nourishment would be called for and that it would be to the best interests of the districts to conserve that nourishment after having gone to the expense of placing it upon the beach. It was also assumed that there would be an economic balance between the supply of more sand and the cost of other methods of protection. The writer stated that the best spacing of the groins depends upon a number of factors. The district will be called upon to supply the necessary amount of nourishment; in the immediate vicinity of the point of supply no groins will be required to regulate the movement of the littoral drift; as the distance from the point of supply increases irregularities in the beach will develop and groins may be necessary to prevent excessive local erosion. Groins are expensive to build, expensive to maintain, unsightly, and cause a wastage of sand. The best spacing will depend upon a balance in the cost of a more generous supply of nourishment, more frequent points of supply, stronger bulkheads, more groins, and the general utility of the improvement finally adopted.

RALPH F. RHODES,[9] M. AM. SOC. C. E. (by letter).[9a]—As stated by the author, the science of beach erosion is new in the United States and comparatively little information dealing directly with the subject is familiar to the average engineer. This paper, therefore, will be a valuable guide to less experienced engineers in the study of the subject, particularly when a problem for determining the cause of beach erosion and making plans for its arrest or prevention at a particular locality is taken up. The author covers his subject comprehensively, and very properly warns the inexperienced engineer against adopting a plan for a particular locality without full consideration of all factors contributing to the destruction of the beach or simply because such a plan has proved effective elsewhere. All plans made for the protection of a beach are, to a certain extent, experimental; but they should be based on the scientific application of all pertinent available data to the problem involved. In the writer's experience, when a beach erosion problem arises and a system of protection works is being planned, usually the plans recommended locally as "certain to do the work best and cheapest" are many and varied in character.

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[8] "The Next Step in Shore Protection," by Elliott J. Dent, *Shore and Beach*, Vol. VI, No. 3, July, 1938.

[9] Senior Engr., U. S. Engr. Dept., Savannah, Ga.

[9a] Received by the Secretary May 17, 1939.

The writer's studies of problems of beach erosion and protection have been confined principally to the coast of Georgia in what the author calls the "South Atlantic Subdivision of the Atlantic Coast." The Georgia coast is part of a long swinging curve extending from the North Carolina capes to the central part of the east coast of Florida. This curve so affects tidal ranges that, while mean ranges are about 3.5 ft and 4.0 ft at the northerly and southerly end of the curve, the value is increased to about 7 ft at Tybee Light, Georgia. The corresponding spring ranges of tide are 4.2 ft at Cape Hatteras, North Carolina, 4.7 ft at Daytona Beach, Fla., and 8.0 ft at Tybee Light. At Cape Hatteras, high water occurs 6 hr and 56 min after the moon's meridian passage, while at Tybee Light it is 15 min later. In general, this tends toward the generation, during a rising tide, of a southerly current along the shore between the two points.

In Georgia, the principal beach erosion problems occur on the ocean shore of the string of islands which form the coast line. These islands are separated from each other by deep sounds scoured out by the swift tidal currents generated by the large tidal ranges on this section of the coast. Some of these sounds are at the mouths of large inland river systems, while the others are the outer ends of large estuaries which project far into the mainland.

The large tidal volumes moving through these sounds necessarily affect the currents along the ocean beaches for a considerable distance on each side of the inlets and, consequently, the sand movement along these beaches. The inlet currents interfere with the general movement of littoral drift toward the south, building it into bars outside of the sounds and pushing the longshore drift out to sea. This robs beaches immediately to the south, of a supply of sand which otherwise they would normally have. These conditions are affected still further by the winds, which vary considerably in direction and intensity. The problems in a beach erosion study in this section are, therefore, vastly more complicated than they are where the beaches are continuous for long distances, the inlets small, and the tidal currents through them moderate.

The author calls attention to the necessity for a study of the changes in the offshore depths. Such a study is important in the vicinity of large inlets, especially where these have built bars immediately to the seaward of the mouth. The extent of such bars affects the location and intensity of the tidal currents entering and leaving the inlet, frequently in such a way as to crowd them in close to the shore line. Under these conditions, rapid erosion of the beaches, especially in the immediate vicinity of the inlet, may be expected due to the fact that all beach material broken down by wave action is carried away rapidly by the accelerated currents along the shore.

As pointed out by Colonel Brown, a knowledge of the geology of the beaches is helpful in solving their erosion problems and designing the protection works. The islands of the Georgia coast were once offshore bars and probably very much more extensive than they are now. Before they became covered with vegetation and trees, sand blew over them and covered extensive areas of marsh between them and the mainland. Since this time, the sea has recaptured a large part, if not all, of the areas occupied by the original islands so that, in

some localities, the present islands consist only of the sand that was carried landward over the bars during prehistoric times. As the ocean shores of these islands erode, a chocolate colored "rock," consisting of a hard mixture of marsh mud and sand, is exposed on the beaches. Although such material may not materially affect the rate of erosion of the beaches, it does affect the design of protection works, at least as to their foundations.

In the study of the shore-line changes and changes in offshore depths, which is an important part of a beach erosion study, a most valuable source of data is found in the U. S. Coast and Geodetic Survey charts. These charts, however, must be used with caution when comparing those of different dates. They usually contain a note such as "Surveys to 1900." This does not necessarily mean that the whole chart was corrected to show conditions as they existed in 1900. What is meant is that the 1900 chart has been revised in certain portions to incorporate data obtained in 1900. Other portions of the chart may show shore lines and offshore depths based on data of a much earlier date. Without knowledge of this fact and just where on the chart the revisions were made, the student may come to the erroneous conclusion, from comparison of charts of different dates, that shore lines and offshore depths in certain areas have remained stable. In such studies it is necessary, therefore, to obtain from the U. S. Coast and Geodetic Survey copies of the field sheets which show the particular area in which revisions have been made, and use them instead of the whole chart. In 1934 the South Atlantic coast line was mapped by aerial methods, and new general surveys were made of offshore depths. The latest editions of these charts are based on these data and show throughout the chart area conditions as they existed in 1934.

As stated by the author, the cost of securing aerial photographs for a section of beach for a small study is prohibitive. This probably refers to photographs made with regular aerial mapping equipment. It may be found, however, that aerial photographs taken by an amateur with ordinary photographic equipment are valuable in supplementing other data. Small planes are often available at reasonable hourly charges, so that flights over a section of beach at relatively frequent intervals may be made at small expense. Photographs so made show changes produced by periods of good and bad weather, and even if the pictures are taken at an angle with the vertical which is not too large, they may be corrected in an ordinary machine during enlargements. The degree of enlargement may also be adjusted, so that their scales are convenient for use in comparing them with maps made by ground methods.

Beach protection works may be divided into two general classes: (*a*) Sea-walls, and (*b*) groins. In certain cases, the two classes may be combined to form a system with a bulkhead parallel to the shore and connected groins perpendicular thereto, extending below the low-water level. Generally speaking, a sea-wall alone is not expected to recapture lost beach area, but only to hold the shore line against further erosion and to enable property in the rear to be developed. The details of construction depend on whether beach area is being lost and deep water is moving shoreward or whether the low-water line is stationary and the high-water line is moving landward. In the former case

the sea-wall must be of heavier construction and its seaward toe well protected from undermining.

The degree of protection ordinarily afforded by a system consisting of a bulkhead and groins is measured, to a large extent, by the amount of sand entrapped and held between the groins and in front of the bulkhead. The groins in themselves offer little protection to the shore line. Their office is to collect and hold sand against the bulkhead, leaving the remainder of the protection to be furnished by the bulkhead structure. Although this structure must be made strong enough to withstand the force of the seas while the sand is collecting in front, it ultimately gives protection only against waves which reach higher elevations than the accumulated sand between the groins.

In designing a system of protective works for a beach which is eroding, full consideration should be given to the length of shore line which should be covered by the protective works. If this is not done, it may be found, after the works are erected, that adjacent sections are beginning to erode which have never before showed signs of cutting back. It may also be found that, as the works are being constructed, the process of erosion is accelerated in adjacent locations where works have not been started. This is because the protective works have trapped sand which was the usual supply of beach building material for the other sections of the beach and the damage has simply been transferred from one locality to another. The susceptibility to such occurrences is greatest in localities where the supply of moving sand for maintaining the beaches is limited. Therefore, the construction program for such conditions should be based on full consideration of probable changes in weather in that part of the beach where no works are yet erected as construction proceeds in the other part. Similarly, consideration should be given to the effect of groins partly or wholly completed on other parts of the beach during sudden storms or other radical changes in the weather.

The author has gone very thoroughly into the details of design of the works for beach protection, and his criteria should not be ignored unless conditions at the locality under study show good and proven reasons therefor. As stated in his first paragraph on the design of protective works, he has assumed shore conditions along straight stretches of beach distant from the influence of an inlet or change in the direction of the shore. For protection against ordinary storm tides and with the land in the rear comparatively low, he specifies that the top of the bulkhead should be placed at an elevation equal to the average height of the highest yearly storm tides plus wave heights. In the vicinity of the mouth of the Savannah River, this would be about 9.5 ft above mean low water, the height of the storm tide, plus 3.5 ft for the wave heights. These two added together give a total of 13 ft, and if the bulkhead consisted of piles with lengths three times this total, 40-ft piles should be used.

In view of Colonel Brown's requirement that the bulkhead should be tied back, so as to insure against failure from pressure from the fill on the back side of the bulkhead, and braced against impact of the breaking waves on the front side, a penetration of 26 ft below low water may be excessive. A bulkhead so designed does not depend for stability on the structural strength of the sheet-

piles at the ground line, the length of these piles below ground being for the most part protection against the wall being undermined. If the worst condition as to scour in front of the bulkhead is assumed, below which no erosion can take place, then the length of piles below this level need be only sufficient to prevent blowing out beneath them from hydrostatic pressure at the rear. A similar analysis for length of pile beneath the level of most severe erosion should be made for groin structures as well. Where brace piles are used, these members are designed for, and intended to take up, all stress due to shock of wave action, and the sheet-piling below the level of most severe erosion is only for the purpose of preventing a blowout beneath the groin due to hydrostatic pressure. In a groin where no brace piles are used, sheeting must take up wave shock also; and in determining its penetration, its length in undisturbed material and the nature of the material must be considered.

On a curving shore approaching a large inlet, the rule laid down by the author for the location and spacing of groins on a straight beach cannot be applied near the inlet. No exact rule which can be used as a guide is known to the writer, but data should be available as to directions and velocities of the currents in and near the inlet at various stages of the tide and at several distances from the shore. How these currents are affected by winds from several directions should also be known. From consideration of these data, it will probably be found that the groin spacing should be less near the inlet than that found correct for a straight section of the beach at a greater distance from the inlet. It will also be found that the pointing of the groins should be about normal to the shore line, the last groin being about at right angles to the axis of the inlet and as far inside as erosion from waves may occur. After reaching this point, it may be found necessary to extend the groin system along the shore into the gorge as far as erosion from inlet currents is taking place. The design for such a groin system, therefore, gradually changes from one based on the specifications laid down by the author for a straight beach to one based on specifications for bank protection against lateral currents.

As Colonel Brown states, there is still need for additional research and study. Well-directed inquiry, however, will disclose considerable useful information of which little is known to the engineer who is engaged in general practice only and to whom the science of beach protection is a comparatively new subject. Such information or references to it can be had for the asking by applying to the Beach Erosion Board mentioned in this paper. However, the engineer who expects to have beach erosion problems presented to him for solution should be continually on the lookout for other information on the subject, especially as to the history of structures already built in other localities where beach, weather, and current conditions are similar to his own.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

DESIGN OF A HIGH-HEAD SIPHON SPILLWAY

Discussion

BY J. C. STEVENS, M. AM. SOC. C. E.

J. C. STEVENS,[5] M. AM. SOC. C. E. (by letter).[5a]—Two outstanding features of this paper are its clarity and brevity. May this discussion be as concise.

It will be of interest to compare the ideological design presented by Mr. Rock with the behavior of the Leaburg[6] and Walterville[7] siphons on which field tests have been made and fully described in the Society's publications.

The writer has set forth the basic principles of siphon behavior in a paper published in 1933.[6] The author's "high-head" siphon corresponds to the case in Fig. 10 of that paper when Section 3 coincides with or lies below Section 2, and the "low-head" siphon when Section 3 lies above (submerges) Section 2.

An important fact in siphon behavior which the author has failed to take into consideration is that the siphon barrel is occupied only in part by water moving down stream and in part by eddies. Fig. 3 of the writer's paper[6] illustrates an idealized concept of the effective and non-effective areas of the Leaburg siphons.

In 1938[8] data were presented to show the eddy area at each cross-section indicating a maximum of 28% of non-effective area. The effect of this phenomenon is to nullify, largely, the author's calculations as to a limiting velocity within the barrel.

The writer will hazard the guess that, if the siphon illustrated by Mr. Rock in Fig. 2 were in use and under observation, the maximum mean velocity would be found, not at the summit section, but somewhere in the lower leg and that its value would be 40 ft per sec or more. Table 2 shows that the maximum

NOTE.—This paper by Elmer Rock, Jun. Am. Soc. C. E., was published in April, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion on the paper.

[5] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[5a] Received by the Secretary May 1, 1939.

[6] "On the Behavior of Siphons," by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 986.

[7] "Siphons as Water-Level Regulators," by J. C. Stevens, *Proceedings*, Am. Soc. C. E., October, 1938, p. 1627.

[8] *Loc. cit.*, Table 4, p. 1638.

TABLE 2.—DATA FROM TESTS OF LEABURG AND WALTERVILLE SIPHONS

	Leaburg Siphon			Walterville Siphon	
	No. 7	No. 6	No. 5	No. 1	No. 2
(*a*) Basic Data:					
Discharge, in cubic feet per second	119	250	470	700	350
Total head, in feet	28.4	27.4	26.4	45.9	45.7
Mean velocity at summit, in feet per second	34.6	28.4	28.5	27.0	29.5
Maximum mean velocity, in feet per second	42.6	44.0	43.0	36.0	41.2
Distance below summit, in feet	7.3	7.2	8.8	29.5	26.2
Ratio of maximum velocity to summit velocity	1.23	1.55	1.51	1.33	1.39
Forebay level, in feet	734.3	734.3	734.6	595.9	595.8
(*b*) For the Crown at the Summit Section:					
Elevation, in feet	735.2	735.8	736.8	598.7	597.4
Pressure head, in feet	−17.0	− 9.6	−10.9	− 9.8	−11.1
Elevation pressure line, in feet	718.2	726.2	725.9	588.9	586.3
Pressure drop from forebay, in feet	16.1	8.1	8.7	7.0	9.5
Energy loss, in feet*	1.6	0.8	0.9	0.7	1.0
Velocity head, in feet	14.5	7.3	7.8	6.3	8.5
Velocity V_1	30.5	21.7	22.4	20.1	23.4
Radius r_1, in feet	3.5	4.0	5.0	5.7	4.5
Radial acceleration $V_1 r_1$	117	87	112	115	105
(*c*) For the Invert of the Summit Section:					
Elevation	734.0	734.0	734.0	595.1	595.0
Pressure head	−30.9	−24.3	−28.8	−30.2	−32.3
Elevation pressure line	703.1	709.7	705.2	564.9	562.7
Pressure drop from forebay	31.2	24.6	29.4	31.0	33.1
Energy loss*	3.1	2.4	2.9	3.1	3.3
Velocity head	28.1	22.2	26.5	27.9	29.8
Velocity V_2	42.5	37.8	41.3	42.4	43.8
Radius r_2	2.0	2.0	2.0	2.0	2.0
Radial acceleration $V_2 r_2$	85	76	83	85	88
Ratio $\frac{V_1 r_1}{V_2 r_2}$	1.35	1.14	1.35	1.35	1.20

* Estimated at $\frac{0.1\,V^2}{2\,g}$; includes entrance, friction, and bend losses, entrance to summit section.

mean velocities in the Leaburg and Walterville siphons were from 23% to 55% greater than the mean velocities in the summit sections and that they occurred well down in the lower leg. During this acceleration the jet must suffer a contraction and leave eddying water around it. The continuity law "$Q = A\,V$" holds only if A is the area of the jet and not that of the barrel.

The author's assumption that the velocity distribution through the summit section follows the free vortex law, wherein the radial acceleration $v\,r$ is constant, is not substantiated by the tests. Table 2 also gives the velocities at the crowns and inverts of the five siphons tested, determined from the piezometer readings at these points. The data may be verified from Table 2 and Figs. 7, 8, and 9 of reference given in Footnote 6, and Table 1 and Fig. 7, of reference given in Footnote 7. Note that the radial acceleration at the crowns is from 14% to 35% greater than at the inverts.

The necessity of admitting water to the summit section through a priming conduit is one factor that may nullify the application of the vortex principle.

The author is correct in stating that the head on a siphon may exceed that of an atmosphere. The highest head that can be utilized is one atmosphere minus vapor pressure of the water plus all entrance, friction, bend, eddy losses, and exit velocity head. On the Walterville siphons the head is nominally 46 ft.

There is no shock or vibration or other evidence of a disconnected water column.

The writer can see no value in contracting the outlet to consume head. If 80 ft of head were available it would be far more practicable to discharge the siphon into an open chute after using 40 ft or so for the siphon head. Open conduits are much cheaper than siphon barrels. This plan was followed at Walterville. In the final stage of reconstruction the tail-water is to be lowered about 7 ft and a chute built from the end of the siphon barrel,[9] to discharge the water well below the water surface, absorbing the energy by an hydraulic jump.

[9] "Siphons as Water-Level Regulators," by J. C. Stevens, *Proceedings*, Am. Soc. C. E., October, 1938, Fig. 6, p. 1634.

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DISCUSSIONS

FLOOD-PROTECTION DATA

PROGRESS REPORT OF THE COMMITTEE

Discussion

BY CHARLES S. BENNETT, M. AM. SOC. C. E.

CHARLES S. BENNETT,[12] M. AM. SOC. C. E. (by letter).[12a]—The compilation of complete data on floods in the form of a "flood inventory" is a highly commendable project. Doubtless there are valuable data of this nature in the files of both public and private agencies which have never been collated or utilized. The very existence of much of this material is unknown to many engineers engaged in flood studies. It is hoped that full use will be made of all of the basic data available from the various sources of information, so that it will be possible to secure, from one reference, data on the storm rainfall, details of run-off, and general information relating to meteorological conditions during specific flood periods. Heretofore, it has been necessary to secure such information from numerous sources, not directly related in their functioning, such as the U. S. Weather Bureau, the Water Resources Branch, U. S. Geological Survey, the U. S. Soil Conservation Service, etc.

No doubt the attempt to compile an inventory of complete flood data, as outlined in the report, will emphasize the fact that many of the basic data on past floods are incomplete. For many water-sheds, particularly the smaller ones, a lack of co-ordination will be found between storm rainfall data and resulting run-off. When the report on storm rainfall of the Eastern United States[13] was being revised in 1936, it was suggested that the inclusion of essential run-off data for the larger storms studied would be of great value. However, it was discovered that in many instances where good rainfall information was available, few, if any, data could be found regarding the resulting run-off, and in some cases the reverse was true. The writer believes that emphasis should be placed upon the need of guaranteeing continuous records

NOTE.—The Progress Report of the Committee on Flood Protection Data was presented at the Annual Meeting, New York, N. Y., on January 18, 1938, and published in January, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1939, by Messrs. Edgar E. Foster, and C. D. Curran.

[12] Engr., The Miami Conservancy District, Dayton, Ohio.

[12a] Received by the Secretary April 21, 1939.

[13] The Miami Conservancy District: *Technical Reports*, Part V (revised 1936).

at suitable existing rainfall and stream-flow gaging stations, rather than the immediate establishment of new stations. Additional stations should be established, of course, when funds and augmented personnel are available.

The Committee (Paragraph 10) calls attention to the fact that changing physical conditions on water-sheds combine to alter run-off rates. Although this is true for many areas, such conditions should encourage, rather than discourage, the continuation of records at existing rainfall and run-off stations on such water-sheds. If data regarding the conditions at a given station, whether changed or unaltered, are published along with the record, the engineer may adjust the data as needed. Surely it would be better to continue a record, even if conditions undergo considerable change, than to break the record or to begin all over at a new station. One of the greatest values of the proposed inventory would be the publication, along with actual flood data for a given storm, of such information as would give the user a means of comparing such data with that for previous and subsequent storms.

It appears that, at last, enough interest has been aroused to secure the provision of funds for the initiation of more detailed studies of flood data, and for the equipment of existing and new rainfall and run-off stations with adequate facilities. If this additional expenditure of funds is to be of permanent benefit, every effort should be made to secure the continuation of the work recently inaugurated.

It is realized that collecting and editing the data necessary for a complete flood inventory will be a relatively slow and tedious undertaking. Nevertheless, the public, and particularly the engineering profession, would be greatly benefited if such data could be made available at the earliest practicable time. This might be accomplished by the publication of the data in installments. This procedure seems to the writer to be preferable to the usual plan of publishing such material only after a research has been completed.

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DISCUSSIONS

POLLUTION OF BOSTON HARBOR

Discussion

By E. Sherman Chase, M. Am. Soc. C. E.

E. Sherman Chase,[7] M. Am. Soc. C. E. (by letter).[7a]—A number of important points relating to the sewerage and drainage of the Boston Metropolitan Area are presented in this excellent paper. Among these points are the distinct limits of capacity life of the drainage systems, in the increasing frequency of overflows as time goes on, the effect of suburban growth and transient populations upon the sewerage problems, the limitations of analytical data, standards of cleanliness for bathing beach waters, the relative economic merits of the proposed extension of the outlets, and the treatment of the sewage without the extension. The writer will limit himself to certain general points of public interest instead of discussing the more technical features of the paper.

From the very beginning of its organization, the Massachusetts State Department of Public Health (and its predecessor, the State Board of Health) has conducted intensive and noteworthy investigations into matters of pollution by sewage, and into methods of sewage treatment. The second annual report of the old State Board of Health, covering the year 1870, contains a most interesting letter from the chairman of the Board, Dr. H. I. Bowditch, entitled "Houses for the People, Convalescent Homes and the Sewage Question"—a somewhat curious combination. In this letter, in which several references are made to his visit to England and the continent in 1870, considerable space is devoted to the controversy, then raging abroad, over the earth closet system *versus* the water carriage system for removal and disposal of excretal matters—a controversy now forgotten.

It is also interesting to note, in light of the consideration given by the authors to the possible production of fertilizer from sludge obtained by treatment of the Boston sewage, the following paragraph from Dr. Bowditch's letter:

Note.—This paper by Arthur D. Weston and Gail P. Edwards, Members, Am. Soc. C. E., was published in March, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[7] Cons. Engr. (Metcalf & Eddy), Boston, Mass.

[7a] Received by the Secretary March 31, 1939.

> "But there is one fatal defect of London and of all American sewage, and that is its waste. Probably there is no such widespread recklessness of spendthrift prodigality anywhere so noticeable among civilized nations as this throwing away of such vast amounts of this most excellent of manures. We take thousands of tons from the earth annually, and totally ignoring Nature's law of economy, which declares that what has been once taken away must be returned again to earth, otherwise the earth itself will become improverished and will refuse to labor for us, I say totally ignoring this law, we squander an immense amount of really valuable property."

Dr. Bowditch was not the first nor the last to feel that way; but after sixty-nine years sanitary engineers have not yet reached the goal of economic use of sewage for its manurial value, as is evidenced by the very tentative way in which the authors discuss the possibility of utilizing sludge for fertilizer.

The interest displayed in the sewage problem by the State Board of Health undoubtedly was one of the reasons, at least, for the construction of the Main Drainage System of Boston and of the Metropolitan Systems. With the introduction of the Cochituate supply of water into Boston in 1848 there had been a marked improvement in sanitary conditions within the city; but objectionable conditions in the adjacent waters resulted from the discharge of sewage from sewers and drains at many points along the water-front. As time went on, the progressive improvements in the collection, interception, and discharge of sewage from the city and the metropolitan districts eliminated the more objectionable conditions, irrespective of the steady growth of population and industry. In fact (as the authors show), based on the criterion of the oxygen balance, the waters of Boston Harbor have ample capacity to assimilate much more sewage than now reaches them.

On the other hand, the fact that solids from the sewers can reach the shores, and the striking evidence afforded by Figs. 3, 4, and 11, show clearly that filthy conditions exist in parts of Boston Harbor. Furthermore, in spite of the difficulty in proving a specific health hazard from polluted waters, other than those used for water supplies, there is a definite trend of public opinion which is resulting in a demand for a standard of cleanliness consistent with ordinary good housekeeping. A similar trend has already been noted for some years in the water-supply field where the public now insists upon supplies of attractive qualities as well as safe from the standpoint of health. The authors have recognized this trend in their conclusions with respect to the undesirability of grease balls and other materials of sewage origin reaching the bathing beaches.

It must be remembered, however, that the correction of conditions of pollution costs substantial sums. In the last analysis the people can have just what they are willing to pay for, and when demands for improved conditions are made, those who make these demands must be prepared and willing to pay their share of the costs. The sanitary improvement of Boston Harbor is a type of public works project that might better be undertaken than many for which the taxpayers' money has been spent in recent years.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

HYDROLOGY OF THE GREAT LAKES
A SYMPOSIUM

Discussion

By A. A. Young, Assoc. M. Am. Soc. C. E.

A. A. Young,[11] Assoc. M. Am. Soc. C. E. (by letter).[11a]—The Symposium on Hydrology of the Great Lakes is an excellent discussion of evaporation from large bodies of water under unusual conditions. The quantitative analysis so closely supported by experimental data is proof of the value of the methods involved. An interesting feature is the shape of the evaporation curve which, contrary to evaporation losses from smaller lakes, shows higher rates of loss in winter than in summer.

This agrees with the fundamental law developed by Dalton (see Equation (5)) which, in brief, states that if all other factors are constant the rate of evaporation is nearly proportional to differences in vapor pressure at the temperature of the water surface and vapor pressure of saturated air at the temperature of the dew-point. When water is warmer than air at less than the saturation point, the difference in vapor pressure is favorable for evaporation, which accounts for the greater losses during winter months. Negative vapor pressures may cause moisture held in the air to return to the water surface although the actual contribution will probably be negligible.

The same conditions were observed by the writer in connection with evaporation studies of the Division of Irrigation of the United States Department of Agriculture in a small covered reservoir in southern California to which relatively warm water was pumped from a near-by well. A narrow belt of screen extending entirely around the superstructure of the reservoir permitted some degree of ventilation but prevented marked air currents at the water surface. Average temperatures of water and of air at the water surface were almost exactly the same during August, with the result that evaporation was only 0.9 in. for the month. Contrasting with this was a

Note.—This Symposium was published in April, 1939, *Proceedings*. This discussion is published in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[11] Associate Irrig. Engr., Div. of Irrig., Bureau of Agri. Eng., U. S. Dept. of Agriculture, Pomona, Calif.

[11a] Received by the Secretary May 26, 1939.

January loss of 2.50 in. under conditions for which the air temperature was 8° less than water temperature. The monthly evaporation curve for the reservoir was an approximate sine curve only from June to November; the next 6 months plotted as an approximate straight line.

Fig. 1(*e*) in the paper by Colonel Pettis shows permanent land losses of 5.9 in. annually, with a maximum monthly loss of 0.9 in. in August. Definition of land losses is given as "the difference between the rainfall *B* and the surface and underground flow into the lake * * *," with *B* indicating precipitation on tributary land areas. Land losses thus include transpiration from all classes of vegetation plus evaporation from moist surfaces.

It would be interesting to know whether the value given to land losses is supported by results of investigations of consumptive use of water by native vegetation and agricultural crops for the area involved. The writer has no data for comparison except estimates by A. F. Meyer[12] of 4 to 12 in. of normal seasonal transpiration for such growth as coniferous and deciduous trees, brush, grains, grass, and agricultural crops. However, investigations of the effect of drought on farm-shelter belts and tree plantations of the northern prairie region in Minnesota, to determine the survival of the principal trees native to the region, following the severe drought of 1934, give an indication of moisture requirements of several tree species.[13] Climatic differences between Minnesota and the Lakes region would appear to be confined principally to differences in humidity which may be favorable to greater consumptive use in the prairie district.

Conditions leading up to the drought of 1934 were in themselves unfavorable to soil moisture. Tree plantings were in relatively good condition prior to 1930. From this time on, subnormal precipitation, coupled with depleted soil moisture, induced severe losses for all tree species. During this period a large percentage of the tree population died for lack of moisture. Much of the area is on the critical line of 20 in. of rainfall. When less rain occurred, tree losses ranged from 22% to 73% of the total number of trees investigated. From this it appears that the minimum water requirement of tree growth on the Minnesota prairies is not far from 20 in. annually. This is considerably different from the land losses for the Lakes area and, despite the difference in localities, leads to some speculation as to whether 5.9 in. annually is the total loss.

Evaporation experiments made by Mr. Hickman as a basis for estimating actual losses from the Great Lakes involved much conscientious work. The evaporation pans were set in water baths in which attempts were made to maintain water temperatures the same as mid-lake temperatures. The close comparison of the pan experiments and the Pettis analysis is remarkable when the difficulties inherent to each method are considered.

Some of the drawbacks of the Weather Bureau type of pan used were overcome largely by the temperature control. The principal objection to the

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[12] "Elements of Hydrology," by A. F. Meyer, John Wiley & Sons, Inc., New York, N. Y., 1917.

[13] "Drouth Damage to Prairie Shelterbelts in Minnesota," by M. E. Deters and Henry Schmitz. *Bulletin No. 329*, Univ. of Minnesota Agr. Experiment Station, 1936.

Weather Bureau pan lies in rapid changes of the water temperatures, which follow air temperatures more closely than in deeper pans set in the ground. The principal advantage of the Weather Bureau pan is in the opportunity of comparing the long series of records obtained from it under a variety of climatic conditions. Records from the pan used in the experiments cannot be so compared because, in effect, it is a floating, rather than a land pan. Records[14] show the relation of evaporation from small reservoirs or large tanks to be about 0.70 of the Weather Bureau pan loss. Under conditions of temperature control, and with the pan surrounded by water, this value would increase and probably approach unity. It would be a matter of interest if Mr. Hickman had stated whether actual pan evaporation was assigned as direct lake evaporation without use of a conversion coefficient.

Since March, 1936, the Division of Irrigation, Bureau of Agricultural Engineering, U. S. Department of Agriculture, has been experimenting with a specially designed ground pan, 2 ft in diameter, with the object of reducing evaporation losses to a point that will not require use of a coefficient. A new principle is introduced in evaporation experiments. By using a screen to intercept a portion of the rays of the sun, comparisons have been made, since 1936, with other pans of which the most important are the Weather Bureau pan and a ground tank, 12 ft in diameter. Evaporation from the screen-covered pan for the 3 yr of record averaged 80% of that from the Weather Bureau pan and 103% of the 12-ft tank loss. The close agreement between losses from the screen-covered pan and the 12-ft tank demonstrates its value as representing lake or reservoir evaporation more closely than any other known type of small evaporation pan. It has been tested variously from coastal to desert climates with close agreement in results under different climatic conditions. It is inexpensive and easily transported and installed. It also has the advantage of using less water than is required ordinarily, and consequently does not require such frequent attention as other pans.

Mr. Hickman's statement (see heading, "Results of Experiments: Temperature Loop") that: "The lake is never at the same temperature throughout, at any one time, and the temperature in the middle may be 20° lower than that recorded near the shore," calls attention to a condition that perhaps has not been given due consideration heretofore. In estimating lake evaporation through use of a conversion coefficient no attention has been paid to the relation of the water surface in the pan to that in the lake. Contrary to lake surfaces, water surfaces in pans have approximately the same temperature for each unit of water area. Pan surfaces are not greatly disturbed by wind, and losses are not increased by waves throwing spray into the air. This all reduces to the fact that evaporation from pans is practically uniform over the pan area, whereas evaporation from lakes is influenced by many factors that have been given little attention in the past and about which relatively little is known.

For some time the writer has been considering factors that influence evaporation from reservoir surfaces. In the West, water is at a premium and its value is increasing. There are many evaporation pan records but few tests

[14] "Evaporation from Different Types of Pans," by Carl Rohwer, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 673.

of the value of such records when applied to large water surfaces. Through the courtesy of the Water Department of the City of Pasadena, Calif., a preliminary investigation was undertaken by the Division of Irrigation, on Morris reservoir, to determine evaporative influences for different parts of the water surface. Instruments placed upon barges recorded air temperature, humidity, and wind movement for different sections of the reservoir surface. Evaporation was measured from a small pan on each barge, and lake temperatures were taken. The results indicated wide differences in evaporation and wind movement and significant differences in temperature and humidity. They were thought to be of sufficient value to justify a more extended study over a much longer period to determine further the distribution of those factors affecting evaporation from the larger water surfaces.

Morris reservoir is a narrow, irregularly shaped body of water, surrounded by abruptly rising mountains that influence wind movement and direction, a condition that also influences temperature and humidity. It is representative of many western mountain reservoirs which are different in shape from those in more open country. There is little doubt that size, shape, depth, and surroundings affect evaporation from such water bodies to a considerable degree, and that such factors at present are not included when reservoir losses are estimated.

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DISCUSSIONS

LATERAL SPILLWAY CHANNELS

Discussion

BY HAROLD ALLEN THOMAS, JR., ESQ.

HAROLD ALLEN THOMAS, JR.,[15] ESQ. (by letter).[15a]—Professor Camp has successfully applied the momentum principle in the analysis of flow in lateral spillway channels of constant width and slope. His ingenious integration of the differential equation provides a general method of calculating the profile of the water surface in a wash-water gutter or gulley. The profiles obtained by his method coincide closely with the measured water surfaces over a large part of the length of the channel. It should be emphasized, however, that the hydraulic theory underlying his analysis does not apply to flow with sharply curving stream-lines and, as a consequence, the differential equation (Equation (17) or (18)) fails in the vicinity of the outfall, as he has shown. In the absence of actual measurements in this region the full success of his analysis is predicated upon the adequacy of empirical rules giving the position of such points as the critical depth in relation to the outfall.

The author's statement (see heading "Critical Depth") that "a close approximation to actual conditions will be obtained if a hydrostatic control section is assumed to exist at a distance up stream from the end equal to three or four times the critical depth" appears to be somewhat arbitrary. In working out a number of surface curves it has been found that it may make a considerable difference whether the "three" or the "four" in the above quotation is used. Moreover, there is no assurance that the effective control section will remain within these limits under all conditions of flow. The writer has experimental evidence that clearly indicates a somewhat wider variation in the position of the effective control section depending upon slope and discharge in the channel than that suggested by the author. Thus it would appear that, to attain the full degree of precision offered by the author's method, it is necessary to have more exact knowledge of the position of the effective control section. Indeed, it has been found that the uncertainty of the location

NOTE.—This paper by Thomas R. Camp, M. Am. Soc. C. E., was published in February, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1939, by Messrs. W. E. Howland, Lewis V. Carpenter and John K. Vennard and Fenner H. Whitley, F. Knapp, and Carl Rohwer.

[15] Instr., San. Eng., Graduate School of Engineering, Harvard Univ., Cambridge, Mass.

[15a] Received by the Secretary May 16, 1939.

of the effective control section frequently obscures the refinement of introducing the friction term into the formulas. Consequently, in the absence of more complete experimental information regarding the curve of the water surface near the outfall, the inclusion of the friction term, in many cases, is wasted effort.

However, in the design of wash-water troughs there may be some question as to the necessity of a complete, meticulous analysis of the flow. From the standpoint of the designer, it usually suffices to know what depth of water to expect in the upper end of the channel under a particular set of conditions. It is seldom necessary in conventional design to compute the entire profile of the water surface. During the past two decades a great many wash-water gutters have been designed in accordance with equations giving the depth of water at the upper end of the channel. In spite of being only semi-rational and containing arbitrary coefficients, these formulas do possess the virtue of simplicity. It seems possible, given the differential equation developed by the author, that a simple formula might be derived for the depth of water at the upper end of the channel (H_0) in terms of the discharge, length, width, and slope, that would eliminate the necessity for successive approximations, as well as the element of trial and error, and yet provide sufficiently precise information for purposes of design.

The foregoing opinion was expressed to the writer by G. M. Fair, M. Am. Soc. C. E., in the hope that an investigation might reveal a suitable basis of simplification. Of course, such simplification, to be of any real value, would have to yield formulas capable of giving more reliable results than semi-empirical formulas used in the past. As a method of approach to the problem, Professor Fair suggested that the integrals in Equation (20) might be evaluated by replacing d by a second degree function of x. In checking various formulas developed according to this scheme with observed water surfaces in wash-water gutters, it was found that the friction term, involving f, made practically no contribution to the precision attained. Since the inclusion of friction forces complicated the formulas considerably, it was believed that a reasonable compromise between precision and simplicity resulted from the omission of the friction term entirely. Therefore, in the equations that follow, friction has been neglected.

The success of the analysis according to Professor Fair's scheme was found to depend upon the selection of the second degree function of x to be substituted for d in the remaining integral of Equation (20). The function finally chosen was a parabola that adheres rather closely to observed water surfaces. The arbitrary constants of the function were so chosen as to cause the parabola to coincide with the actual surface at $x = 0$ and $x = x_c$ and, further, to possess a slope of S when $x = 0$ as is required by Equation (18). This function of d in terms of x is as follows:

$$d = H_0 + S\,x + \frac{(d_c - H_0 - S\,x_c)\,x^2}{x_c^2} \quad \text{.............(53)}$$

When this value is placed in Equation (20) and if $f = 0$, the following equation may be written:

$$\frac{x^2}{d} + \frac{g\,b^2\,d^2}{2\,q^2} = \frac{S\,g\,b^2}{q^2}\int\left[\,H_0 + S\,x + \frac{(d_c - H_0 - S\,x_c)}{x_c^2}\,x^2\,\right]dx + c \;..(54)$$

Since $\frac{g\,b^2}{q^2} = \frac{x_c^2}{d_c^3}$, and since $d = H_0$ when $x = 0$, the integrated equation may be written in the following form:

$$\frac{x^2}{d} + \frac{x_c^2}{2\,d_c^3}\,(d^2 - H_0^2) = \frac{S\,x_c^2}{d_c^3}\left[H_0\,x + \frac{S\,x^2}{2} + \frac{(d_c - H_0 - S\,x_c)}{3\,x_c^2}\,x^3\right] \quad (55)$$

Substituting $x = x_c$ and $d = d_c$ and solving for H_0, the final formula is

$$H_0 = \sqrt{2\,d_c^2 + \left(d_c - \frac{S\,x_c}{3}\right)^2} - \frac{2}{3}\,S\,x_c \quad (56a)$$

In comparing values of H_0 given by Equation (56*a*) with those obtained by measurement of the water surface over a wide variety of slopes and discharges, a rather close agreement was observed. In addition, it was found that the degree of precision attainable by this formula was not appreciably affected by taking x_c equal to the entire length of the channel (L), and d_c as calculated from the entire discharge by the formula $d_c = \sqrt[3]{\frac{Q^2}{g\,b^2}}$. Accordingly, Equation (56*a*) becomes

$$H_0 = \sqrt{2\,d_c^2 + \left(d_c - \frac{S\,L}{3}\right)^2} - \frac{2}{3}\,S\,L \quad (56b)$$

For $S = 0$, Equation (56*b*) reduces to

$$H_0 = 1.73\,d_c \quad (57)$$

A number of rapid sand filter plants are designed or operated in such a manner that the wash-water does not discharge freely from the gutters to the central gulley. In this case the height of the water surface in the lower end of the channel exceeds that occurring with free discharge, and is determined by hydraulic conditions down stream. Therefore, if the hydraulic control lies beyond the gutter, the use of equations based on free discharge may involve considerable error. It is of interest to the designer to know what depth of water to expect in the upper end of the wash-water gutter for various depths in excess of the critical depth at the lower end of the channel. This depth of water (H_0), at the upper end of the channel, is given by the following formula, which was derived in the same manner as Equation (56*a*), for the condition $x = L$, $d = d_L$ (where d_L is the measured or expected depth of water at the lower end of the channel).

$$H_0 = \sqrt{\frac{2\,d_c^3}{d_L} + \left(d_L - \frac{S\,L}{3}\right)^2} - \frac{2}{3}\,S\,L \quad (58)$$

and for $S = 0$ this reduces to:

$$H_0 = \sqrt{\frac{2\,d_c^3 + d_L^3}{d_L}} \quad (59)$$

It is to be noted that Equation (56*b*) may be obtained from Equation (58) by making $d_L = d_c$.

For the convenience of the designer the relationship given by Equation (58) (including the special cases covered by Equations (56*b*), (57), and (59)) could be easily expressed in the form of a diagram.

The order of agreement obtained using Equations (56*b*) and (59) with the method of the author for the various sets of experimental measurements included in his investigation is indicated in Table 5. The comparison is based on the quantity $H_0 + L\,S$, since this represents the actual over-all head in the channels.

The average error in Table 5 is 1.5% and it may be noticed that the largest discrepancy, occurring in the Division Avenue Plant, is only 3.8 per cent. This should serve to demonstrate that the divergence between the two methods is quite small; in fact, it may actually fall within the limits of experimental error inherent in the measurement of water depths and discharge.

TABLE 5.—COMPARISON OF VALUES OF $H_0 + L\,S$ OBTAINED BY THE AUTHOR WITH THOSE CALCULATED USING EQUATIONS (56*b*) AND (59)

Test (1)	Discharge, Q, in cubic feet per second (2)	$H_0 + L\,S$ in feet given by the author (3)	$H_0 + L\,S$ in feet using Equations (56b) and (59) (4)	Percentage error‡ (5)
Detroit, Mich., Experiments—				
Run No. 1	10.91	1.84	1.82*	−1.1
Run No. 2	9.05	1.66	1.63*	−1.8
Cleveland, Ohio, Experiments Division Avenue Plant	55.6	4.44	4.25†	−3.8
Baldwin Filtration Plant	73.6	5.24	5.21*	−0.6
Massachusetts Institute of Technology Experimental Lateral Spillway Channel	1.237	0.80	0.794†	−0.7

* Computed by Equation (56b). † Computed by Equation (59). ‡ $\dfrac{\text{Column (4)} - \text{Column (3)}}{\text{Column (3)}} \times 100$.

In order to investigate further the precision offered by Equations (56*b*), (57), (58), and (59) under a wide range of channel slopes and discharges, a small lateral spillway channel was constructed and tested in the Gordon McKay Engineering Laboratory of Harvard University, at Cambridge, Mass. This channel, made of cypress, was 5 ft 11 in. long and 4 in. wide, with level top edges and bottom of adjustable slope. Experience has demonstrated that the results obtained from model channels of this size and type may be translated to pertain to structures of larger dimensions by application of the ordinary Froude law of hydraulic similitude. Investigations with this channel proved to be satisfactory in every respect.

A summary of the degree of accordance between Equations (56*b*) and (57) and experimental information acquired from measurement of the water surface at the upper end of the channel under various conditions of slope and flow with free discharge at the outfall is presented in Table 6. In Table 6, as before, the comparison is based upon the quantity $H_0 + L\,S$ for convenience in comparing tests of different slope.

TABLE 6.—COMPARISON OF VALUES OF $H_0 + L\,S$ OBTAINED (1) BY MODEL EXPERIMENTS UNDER CONDITIONS OF FREE DISCHARGE AND (2) BY CALCULATION USING EQUATIONS (56b) AND (57)

Slope (1)	Discharge, Q, in cubic feet per second (2)	$H_0 + L\,S$ in inches measured in channel (3)	$H_0 + L\,S$ in inches, calculated using Equations (56b) and (57) (4)	Percentage error* (5)
0.0	0.055	2.04	1.97	−3.4
0.0	0.0703	2.30	2.32	−2.9
0.0	0.0876	2.70	2.68	−0.7
0.0208	0.0546	2.21	2.21	0.0
0.0208	0.0652	2.39	2.44	+2.1
0.0208	0.1033	3.19	3.23	+1.3
0.0313	0.072	2.92	2.76	−5.5
0.0313	0.0962	3.34	3.32	−0.6
0.0313	0.1097	3.63	3.50	−3.6

$*\ \dfrac{\text{Column (4)} - \text{Column (3)}}{\text{Column (3)}} \times 100.$

In Table 6 the average absolute error is only 2.2%, and the largest error is 5.5%, occurring with a slope of 0.0313 and a relatively small discharge. Although these errors are small, it should be pointed out that, with increasing slopes, the velocities in the channel become larger and the friction term becomes of increasing importance. Hence, it is not expected that Equations (56a), (57), (58), and (59), in which this term is neglected, will apply indefinitely. Using these equations with slopes greater than 0.055, errors in excess of 15% have been found. Such slopes, however, involve an uneconomical channel design; it will be shown that Equations (56a), (57), (58), and (59) are valid for the range of slopes occurring in economically designed channels.

In Table 7 is presented a summary of experiments in which the hydraulic control section did not occur in the channel. In these tests the depth of the water in the lower end of the gutter exceeded the critical depth. To show the extent to which the water was backed up in excess of the critical depth, the quantities d_c and d_L have been included in Table 7. The depths $H_0 + L\,S$ measured at the upper end of the channel are compared with corresponding depths in this region calculated with Equations (58) and (59).

TABLE 7.—COMPARISON OF VALUES OF $H_0 + L\,S$ OBTAINED (1) BY MODEL EXPERIMENTS IN WHICH d_L EXCEEDED d_c, AND (2) BY CALCULATION USING EQUATIONS (58) AND (59)

Slope (1)	Discharge, Q, in cubic feet per second (2)	d_L in inches, measured (3)	d_c in inches, calculated (4)	$H_0 + L\,S$ in inches, measured (5)	$H_0 + L\,S$ in inches, calculated using Equations (58) and (59) (6)	Percentage error* (7)
0.0	0.0243	1.34	0.658	1.44	1.51	+4.9
0.0	0.0464	2.12	1.03	2.34	2.38	+1.7
0.0	0.0628	2.61	1.24	2.92	2.95	+1.0
0.0313	0.0123	2.45	1.36	3.14	2.99	−4.8
0.0313	0.1000	3.18	1.69	3.69	3.74	+1.4

$*\ \dfrac{\text{Column (6)} - \text{Column (5)}}{\text{Column (5)}} \times 100.$

The average absolute error in Table 7 is 2.8%, and the maximum error does not exceed 5 per cent. This is in agreement with the order of precision attained in the other experiments. Therefore it can be concluded that Equations (56a), (56b), (57), (58), and (59) give sufficiently precise information for practical purposes. Indeed, under many conditions, the error involved in using these equations is of a considerably smaller magnitude than the errors that result in practice from incorrectly estimated discharges and improperly leveled weir crests.

Economics of Wash-water Gutter Design.—An interesting application of Equation (56b) is the determination of the slope of the most economical wash-water gutter, assuming that Q, b, and L are specified by the rate of back-wash and filter-bed dimensions. The cost of a wash-water gutter may be taken as the cost of the sides and bottom and, in addition, certain fixed costs that do not depend upon the dimensions of the channel. This may be stated mathematically as follows:

$$C = 2\,K_s\,L\left[\frac{H_0 + (H_0 + L\,S)}{2}\right] + K_b\,b\,L + K_f \quad \text{........ (60)}$$

in which C = cost of channel; K_s = cost of sides per unit area; K_b = cost of bottom per unit area; and K_f = fixed costs (assumed to be independent of dimensions of the channel). Substituting the value of H_0 from Equation (56b), Equation (60) becomes

$$C = K_s\,L\left[2\sqrt{2\,d_c^2 + \left(d_c - \frac{S\,L}{3}\right)^2} - \frac{S\,L}{3}\right] + K_b\,b\,L + K_f \quad \text{... (61)}$$

Since Q, b (hence d_c), L, K_s, K_b, and K_f are constants, the cost of the channel depends only upon the value of S. Differentiating Equation (61) with respect to S and setting the derivative equal to zero:

$$2\left(d_c - \frac{S'\,L}{3}\right) = -\sqrt{2\,d_c^2 + \left(d_c - \frac{S'\,L}{3}\right)^2} \quad \text{.......... (62)}$$

Solving for S', the slope corresponding to minimum cost of channel,

$$S' = 0.55\,\frac{d_c}{L} = \frac{0.55}{L}\sqrt[3]{\frac{Q^2}{g\,b^2}} \quad \text{.................... (63)}$$

From Equation (63) it may be seen that the optimum slope increases with discharge and decreases with the length and width of the channel. For average values of Q, L, and b used in the United States the most economical slope, according to Equation (63), usually falls between 0.0125 and 0.0175.

Conclusions.—It has been shown that the author's elaborate treatment of flow in lateral spillway channels, for practical purposes, may be simplified into elementary formulas or diagrams that show a remarkable agreement with experimental measurements. These formulas lend themselves to an analysis of the economics of wash-water gutter design in which a rule is developed for the determination of the optimum slope of the channel.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

DESIGN OF CIRCULAR CONCRETE TANKS

Discussion

BY MESSRS. FRANK J. MCCORMICK, H. B. MUCKLESTON, ROBERT B. B. MOORMAN, DANA YOUNG, L. J. MENSCH, JENS EGEDE NIELSEN, LLOYD S. DYSLAND, BASIL SOUROCHNIKOFF, AND REGIS F. FEY

FRANK J. MCCORMICK,[11] JUN. AM. SOC. C. E. (by letter).[11a]—The author's repeated statement that he has presented a strictly rigorous analysis is open to question, because an analysis based on the assumption of ring stress uniformly distributed over the thickness of the wall is not strictly rigorous except for very thin walls. However, for a wall of homogeneous elastic material, it can be shown that his assumption is justified, in the design of tanks of usual proportions, by making use of Lamé's theory of thick-walled cylinders as described by Professor Timoshenko.[12, 13] He shows that the ring stress in the wall of a circular cylinder subjected to internal pressure is given by the expression

$$f_s = b \times \frac{a^2\,p}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right) \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

in which: a = the internal radius of the tank; b = the external radius of the tank; and r = the radius at which f_s is given by Equation (27). Equation (27) indicates maximum ring stress at the inner surface of the wall, and it is easy to determine that the ratio of the maximum stress to the minimum stress at any section is given by the expression

$$\frac{\text{Maximum } f_s}{\text{Minimum } f_s} = \frac{a^2 + b^2}{2\,a^2} \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

For the tank used in the author's example, $\frac{b}{a} = \frac{35.75}{35} = 1.0214$; and,

NOTE.—This paper by George S. Salter, M. Am. Soc. C. E., was published in March, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1939, by Messrs. Carl A. Rosengarten, and I. K. Silverman.

[11] Draftsman, Bridge Dept., State Highway Comm., Ames, Iowa.

[11a] Received by the Secretary April 13, 1939.

[12] "Strength of Materials," by S. Timoshenko, Part II, p. 531.

[13] "Über die Spannungsverteilung in zylindrischen Behälterwänden," by H. Reissner, *Beton und Eisen*, B. 7, H. VI, 1908, p. 150.

$\frac{\text{Maximum } f_s}{\text{Minimum } f_s} = 1.021$, which indicates a variation of only 2.1% in the stress across any section. For $\frac{b}{a} = 1.1$ the variation is found to be only $10\frac{1}{2}$ per cent.

The position of the resultant, S, may be derived by writing the moment of f_s about the center of the tank,[14] integrating throughout the thickness of the wall, and dividing this quantity by the total stress, S. Thus,

$$r_s = \frac{\int f_s\, r\, dr}{\int f_s\, dr} = b \times \frac{\frac{1}{2}\left[1 - \left(\frac{a}{b}\right)^2\right] + \log_e \frac{b}{a}}{\frac{b}{a} - \frac{a}{b}} \qquad \text{(29)}$$

in which r_s = the radius to the resultant total stress, S. In the author's example, $\frac{b}{a} = 1.0214$ and from Equation (29) $r_s = 34.3737$ ft. The distance from the center of the wall to the point of action of the resultant stress, S, is then only 0.0156 in., or 0.0017 t. The tacit assumption by the author, that there is no moment on a vertical section and that the stress is uniformly distributed across the section, may thus be justified.

For a tank wall the flexural rigidity is more accurately represented by the expression $\frac{E\, t^3}{12\,(1 - \mu^2)}$ (in which μ = Poisson's ratio) rather than by the author's[15] term $\frac{E\, t^3}{12}$. Because of the uncertainty involved in the evaluation of this quantity for reinforced concrete, and because the value usually chosen for μ is small (less than 0.25) probably no serious error results from the use of the simpler expression.

After evaluation of the constants in Equations (14) by the introduction of the boundary condition, $y = 0$ at $x = 0$, the author later discovers no ring stress at the base of the wall. This, he concludes, indicates that the pressure at this point is resisted entirely by the cantilevers. Any other conclusion would be much more alarming, as this result is implicit in the boundary condition just mentioned, and merely indicates the consistency of the intervening mathematics.

It should be mentioned that Equation (10), and its solution, Equation (11), were published by C. Runge in 1904.[16] H. Reissner extended the solution to include walls of variable thickness in 1908.[13] The latter work, reproduced and further extended by V. Lewe,[17] includes tables and curves to aid in the computation of the radial displacements, ring stresses, and cantilever moments. In part the tables and curves are equivalent to those given by the author in Figs.

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[14] "Über die Spannungsverteilung in zylindrischen Behälterwänden," by H. Reissner, *Beton und Eisen*, B. 7, H. VI, 1908, p. 151.

[15] "Strength of Materials," by S. Timoshenko, Part II, p. 475; see also "Applied Elasticity," by John Prescott, p. 391.

[16] "Über die Formänderung eines zylindrischen Wasserbehälters durch den Wasserdruck," by C. Runge, *Zeitschrift für Mathematik und Physik*, B. 51, 1904, p. 254; see also "Die graphische Statik der Baukonstruktionen," by H. Müller-Breslau, B. 2, II Abt., Aufl. 2, 1925, p. 207.

[17] "Die statische Berechnung der Flüssigkeitsbehälter," by V. Lewe, *Handbuch für Eisenbetonbau*, B. 5, 3 Aufl., 1923, pp. 87–96.

3 and 4; but they also include data pertaining to tanks with walls of triangular section as well as to those with walls of constant thickness. In this work, furthermore, may be found a rather incomplete table of constants for use in the analysis of shallow circular tanks with walls of trapezoidal section.

H. B. MUCKLESTON,[18] M. AM. SOC. C. E. (by letter).[18a]—Stresses in a circular reinforced concrete tank are analyzed by an interesting method in this paper. In the course of this analysis, the author makes certain simplifying assumptions without which the resulting differential equation probably could not be integrated:

(1) Implicit in Equations (3) and (4), that the area of the ring steel is uniform from bottom to top of the wall;

(2) Implicit in Equations (9) and (17), that the moment of inertia of the elementary cantilevers is uniform from top to bottom (which means that the steel areas in both faces are uniform);

(3) Implicit in Equation (9), that deformation of the cantilever elements from shear may be neglected;

(4) That E and E_0 may be considered as equal; and,

(5) That Poisson's ratio need not be considered.

Although the last three of these may be of small importance, the first two are fundamental and must be closely realized in design or the entire analysis becomes null.

In Table 1, the author has adjusted the ring-steel areas to the computed ring stresses on a basis of 12 000 lb per sq in. If the tank were built to these areas, the computed ring stresses could not exist since they were computed on the hypothesis that the steel area was uniform. In fact if they did exist, the deflection y would be uniform from bottom to top, which is contrary to the hypothesis.

Either the ring steel and cantilever steel must be kept uniform (in which case the analysis is applicable), or they may be variable, in which case the analysis does not apply and a solution must be sought by some trial-load method.

If they are kept uniform, there is a question whether a tank so designed will show any economy over a tank of similar dimensions designed according to the simple cylinder hypothesis. In the example the maximum ring steel area is 0.77% of the concrete area. If the hypothesis is to apply, the same proportion must be used for the entire height. The ring steel will then be 0.77% of the entire volume of concrete in the wall. To this must be added the vertical steel which will scarcely be less than 0.75% of the volume. This is a total steel for the wall, amounting to 1.52% of the wall volume.

Using the same wall thickness and steel stress, the ring steel in a design by the simple cylinder hypothesis would require about 1.07% of the wall volume, to which about 0.18% must be added for vertical spacing rods. The difference, for a tank of these dimensions, is not far from 2 700 lb of steel. Moreover, the

[18] Cons. Engr., Vancouver, B. C., Canada.

[18a] Received by the Secretary April 14, 1939.

steel in the latter case would be cheaper to erect, and the wall thickness might well be somewhat less.

It might be argued that leaving the cantilever action out of consideration will result in ring cracks near the base of the wall. It is questionable if experience would bear out this contention. Many tanks have been built on that basis and it is not on record, so far as the writer knows, that even a large proportion of them show such cracks. If they are feared, the steel in the lowest foot might be computed on a basis of, say, 8 000 lb per sq in. That would reduce the outward deflection to a point where the shear resistance of the concrete could take care of it easily. In the case in question, the outward movement, if there were a smooth, frictionless joint at the base, would be only about 0.0117 in.

ROBERT B. B. MOORMAN,[19] ASSOC. M. AM. SOC. C. E. (by letter).[19a]—This paper is a very worth while contribution to American engineering literature on the subject of circular concrete tanks. In England W. S. Gray has published two books[20, 21] on the design of reinforced concrete tanks and water-towers. The information on the analysis of circular tanks in these two books was obtained from an earlier publication[22] by H. Carpenter explaining the method by H. Reissner.[23]

Figs. 2, 3, and 4, however, are more comprehensive. According to Reissner's theory all tanks with equivalent K-values $\left(K = \frac{12\,h^4}{r^2\,t^2}\right.$, in which r = radius of inside of tank$\left.\right)$ have similar load-distribution curves. This K-value is similar to the author's θ^4. The author bases his curves on equivalent θ-values, whereas the curves presented by Gray are based on equivalent K-values.

Equation (17) would be understood more clearly if it were written

$$\theta = \frac{h}{\pi}\sqrt[4]{\frac{3}{r^2\,t^2}}\,\pi = h\sqrt{\frac{0.176}{r\,t}}\,\pi \qquad (30)$$

This equation may be simplified further by letting $R\,h = r$ and $T\,h = t$ and writing the expression for θ as

$$\theta = \frac{0.419}{\sqrt{R\,T}}\,\pi \qquad (31)$$

The work involved in computing θ by use of Equation (30) or (31) would probably be about the same when reviewing a tank of given dimensions. However, Equation (31) is more flexible in that only relative dimensions are needed to compute the value of θ.

Using the data of the author's example, the bending moment (both positive and negative) and shear are computed as follows:[20] Since the curves presented

[19] Asst. Prof., Civ. Eng., Univ. of Missouri, Columbia, Mo.

[19a] Received by the Secretary April 10, 1939.

[20] "Reinforced Concrete Reservoirs and Tanks," by W. S. Gray, Concrete Publications Limited, London, England, 1931.

[21] "Reinforced Concrete Water Towers, Bunkers, Silos and Gantries," by W. S. Gray, Concrete Publications Limited, London, England, 1933.

[22] *Concrete and Constructional Engineering*, Concrete Publications Limited, London, England, Vol. 22, April, 1927, p. 237, and Vol. 24, June, 1929, p. 345.

[23] "Handbuch für Eisenbetonbau," Vol. V.

by Gray are for values of K equal to 0, 10, 100, 1 000, and 10 000, only a rough check can be expected. For this example

$$K = \frac{12\,(12.5)^4}{(35)^2\,(0.75)^2} = 425.$$

Upon sketching a curve for $K = 425$,[24] values are obtained for the coefficient, C_t, by use of which the ring tension can be computed in pounds per foot of wall height (see Table 2).

TABLE 2.—COMPUTATION OF RING TENSION

Description	Tenth-Points, with Point 1.0 at the Top										Average
	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	
Coefficient, C_t	0.13	0.18	0.27	0.31	0.36	0.37	0.33	0.25	0.15	0.06	0.241
Ring tension, in pounds per foot of wall height	3 560	4 920	7 390	8 480	9 850	10 120	9 030	6 840	4 100	1 640	....

A coefficient[25] $C_{-m} = 0.036$ for the negative moment gives $-M = 0.036 \times 62.5 \times (12.5)^3 = 4\,390$ ft-lb per ft. The value of the positive moment is taken as one-fourth to one-third of the negative moment or, using an average of this range, $M = 0.29 \times 4\,390 = 1\,273$ ft-lb per ft.

The shear can be obtained by subtracting the portion of the horizontal load carried by the rings from the total horizontal load; or, $V = (0.5 - 0.241)$ $62.5 \times (12.5)^2 = 2\,539$ lb per ft. The use of the coefficient 0.241 is not quite proper since it is the average coefficient for the ring tension at the top of the tenth-points. These values of the positive and negative moments and shear agree fairly well with the author's values.

Although, in practically all cases, Equations (18) and (19) are satisfactory, the more exact expressions are

$$f_c = \frac{C\,E_c\,n\,p}{1 + (n-1)\,p} \quad \text{..........(32)}$$

and

$$f_s = \frac{T}{A_c\,[1 + (n-1)\,p]} \quad \text{..........(33)}$$

That is, $f_c = 65$, $f_s = 86$, and the total stress is 151 lb per sq in.

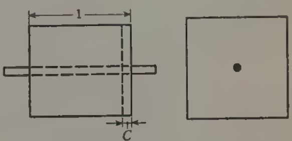

FIG. 7

A derivation of the expression for unit stress in the concrete due to contraction (Equation (32)), which is similar to Equation (19), follows: A steel bar, embedded in concrete, is shown in Fig. 7. The cross-section of the steel is $A\,p$ and of the concrete is $A\,(1 - p)$. In a unit length the concrete tends to shorten a distance

[24] From curves by Carpenter; "The Calculation of Cylindrical Tanks with Rectangular, Triangular or Trapezoidal Wall Sections," by H. Carpenter, *Concrete and Constructional Engineering*, Concrete Publications Limited, London, England, June, 1929, Fig. 2, p. 347.

[25] Selected from curves; *loc. cit.*, Fig. 3, p. 348.

C, which is the coefficient of contraction of the concrete. The deformation in the steel must equal the deformation in the concrete, or

$$\frac{f_s}{E_s} = C - \frac{f_c}{E_c} \qquad (34)$$

In order that the system shall be in equilibrium the summation of the horizontal stresses must equal zero, or

$$A\, p\, f_s = A\,(1 - p)\, f_c \qquad (35)$$

from which

$$f_s = \frac{(1 - p)}{p} f_c \qquad (36)$$

is obtained. From Equation (34):

$$f_s = C\, E_c\, n - f_c\, n \qquad (37)$$

Equation (32) is derived by combining Equations (36) and (37) and solving for f_c.

Corrections for *Transactions:* Equation (9b), for slope, should read "$\frac{dy}{dx} = \theta$"; and, in Figs. 2 and 3, ordinate values of θ are in terms of π; see also *Proceedings*, May, 1939, p. 908.

DANA YOUNG,[26] ASSOC. M. AM. SOC. C. E. (by letter).[26a]—The general analysis of the problem stated in this paper has been developed thoroughly by various European engineers and is presented in standard references such those of Pöschl,[27] Flügge,[28] Love,[29] and Löser and Lewe.[30] However, the work done by the author in determining the constants defined in Equations (14) is of great value in designing relatively shallow tanks.

Any method of analysis is based on various assumptions which the designer must bear in mind. In this regard it may be of interest to discuss several points in the paper. In Equation (3), the unit tensile stress in the concrete is found by dividing the total stress by the cross-sectional area of the ring. It would be more correct to divide by the area of the transformed section $(1 + n\,p)\,t\,dh$ as the author finally does in Equation (19). Equation (9), which is the formula of flexure for a beam, does not apply strictly to a strip of a plate or shell since the adjoining strips restrain the free transverse deformations. As shown in various references,[31] for the case of a plate or shell the equation of flexure is

$$\frac{E\,I}{1 - \nu^2}\frac{d^2y}{dx^2} = M \qquad (38)$$

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[26] Assoc. Prof. of Civ. Eng., University of Connecticut, Storrs, Conn.

[26a] Received by the Secretary April 21, 1939.

[27] "Berechnung von Behältern," by Th. Pöschl, Berlin, Germany, 1926.

[28] "Statik und Dynamik der Schalen," by W. Flügge, Berlin, Germany, 1934.

[29] "A Treatise on the Mathematical Theory of Elasticity," by A. E. H. Love, Cambridge England, 1927.

[30] "Behälter," by B. Löser and V. Lewe, in Emperger's *Handbuch für Eisenbetonbau* Third Edition, Vol. 9, Berlin, Germany, 1934.

[31] "Strength of Materials," by S. Timoshenko, New York, N. Y., Part II, 1930, pp. 475 and 517; see also Footnote References 27–30.

in which ν is Poisson's ratio. Hence, using Equation (9) is equivalent to assuming that Poisson's ratio is zero. These corrections would not affect the analysis except that the value of the constant β would then be given by

$$\beta = \sqrt[4]{\frac{E_0\, t\,(1 + n\,p)\,(1 - \nu^2)}{4\,r^2\,E\,I}} \quad \text{.................(39)}$$

The numerical effect of these corrections would be small except in unusual cases. Furthermore, E_0 will equal E in most cases.

The solution of Equation (10*b*) given by the author is based on the assumption that the tank is of constant cross-section and has the same reinforcement throughout its height; that is, that t and I are constant. Small variations in the percentage of reinforcement probably will not change the results seriously, but caution should be used in any design in which t and I vary considerably. The analysis for such conditions is more complicated, of course, but is well treated in the references of Pöschl[27] and Flügge.[28]

As is clearly stated by the author, the constants determined by Equation (14) are for the case in which the base is completely fixed. The designer must consider carefully whether or not this condition is realized in the tank being designed. If the base is not perfectly restrained against rotation of a vertical element, the negative moment at the base will be less than that calculated, but the values of the maximum ring tension and positive moment will be greater.

It may be well to note that the condition which the author assumes at the top where x equals h—namely, that the shear and moment are zero there—is true only if the water level is exactly at the top of the tank. If this is not so, the part of the tank above the water level exerts a restraint on the deformations at the water level. This effect can be taken care of in the evaluation of the constants, although the numerical work involved would be tedious.

The analysis of a circular tank is simplified when the depth is great, or more accurately, when θ is large (say, greater than π). In this case, it may be assumed that $A = B = 0$, or (what is the same thing) that $C_1 = C_2 = 0$. Mathematically this follows from the fact that the term in Equation (11) containing these constants represents a damped harmonic wave which starts at the upper boundary and is so rapidly damped that it has little effect on the conditions at the base. Physically, this may be seen from the fact that, in a deep tank, the conditions at the upper boundary will have little effect on those at the base. This is clearly shown in Fig. 2, in which, for values of θ greater than about $1.2\,\pi$, the values of the constants C_1 and C_2 are practically zero. Using this assumption, Equations (14) give $A = B = 0$; $C = -F\,\theta$; and, $D = F\,(1 - \theta)$. Then, further assuming a homogeneous tank of constant thickness and neglecting Poisson's ratio $\left(\text{in which case } \theta = h\,\sqrt[4]{\frac{3}{r^2\,t^2}}\right)$, the moment at the base is found by substituting the foregoing values for the constants in Equation (13*a*). After simplifying, this step reduces to

$$(M)_{x=0} = E\,I\left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{\theta - 1}{2\,\theta^3}\,w\,h^3 \quad \text{..............(40)}$$

which checks closely with Fig. 3 for values of θ greater than π. Similarly the shear at the base is found to be

$$(V)_{x=0} = E\,I\left(\frac{d^3y}{dx^3}\right)_{x=0} = \frac{1 - 2\,\theta}{\theta^2}\,\frac{w\,h^2}{2}\ldots\ldots\ldots\ldots(41)$$

which checks Fig. 5 for large values of θ. The ring tension for this case is given by

$$T = \frac{E\,t}{r}\,y = w\,h\,r\left\{\left(1 - \frac{x}{h}\right) - e^{-a}\left[\cos\theta\frac{x}{h} + \left(1 - \frac{1}{\theta}\right)\sin\theta\frac{x}{h}\right]\right\}\ldots(42)$$

in which $a = \theta\,\frac{x}{h}$. Numerical values computed from Equation (42) for the case, $\theta = 1.5\,\pi$, are given in Table 3.

TABLE 3.—RING TENSION COEFFICIENT C_t FOR $\theta = 1.5\,\pi$

$\frac{x}{h}$	FIXED BASE		PINNED BASE
	From Fig. 4	Computed from Equation (42)	Computed from Equation (46)
1.0	0.02	0.007	0
0.8	0.23	0.229	0.219
0.6	0.44	0.442	0.456
0.4	0.53	0.533	0.647
0.2	0.32	0.323	0.571

To show the effect when the wall at the base is not perfectly fixed, it is interesting to investigate the case in which the wall is pinned at the base. Then the results for partial restraint will lie between those for this case and those for the fixed base. Again assuming that $A = B = 0$, the constants C and D are determined from the condition that the deflection and moment at the base of the wall are zero. Substituting these conditions in Equations (11) and (13b), the values of the constants C and D are found to be $C = -F\,\theta$ and, $D = 0$. Then the bending moment is given by

$$M = E\,I\frac{d^2y}{dx^2} = -2\,E\,I\,F\,\theta\,\beta^2\,e^{-\beta x}\sin\beta\,x\ldots\ldots\ldots\ldots(43)$$

in which the negative sign indicates that the bending produces compression on the inside fibers; that is, the moment is positive according to the convention used by the author. The maximum numerical value for this moment occurs where $x = \frac{\pi}{4\,\beta}$. Substituting this value of x in the moment equation gives the following for the maximum positive bending moment

$$M = \frac{0.161}{\theta^2}\,w\,h^3\ldots\ldots\ldots\ldots\ldots\ldots(44)$$

Values of this positive moment are greater than the positive moment in the case of a tank with a fixed base.

The shear at the base is given by

$$(V)_{x=0} = E\,I\left(\frac{d^3y}{dx^3}\right)_{x=0} = -\,2\,E\,I\,\beta^3\,F\,\theta = -\,\frac{1}{\theta}\,\frac{w\,h^2}{2} \quad\ldots\ldots(45)$$

These shears are less than those for the case of a fixed base.

The ring tension for the case of the wall pinned at the base is given by

$$T = \frac{E\,t}{r}\,y = w\,h\,r\left[\left(1 - \frac{x}{h}\right) - e^{-\alpha}\cos\theta\,\frac{x}{h}\right] \quad\ldots\ldots\ldots(46)$$

The maximum ring tension for this case is greater than for a fixed base. For comparison, the ring tension coefficients as computed from Equations (42) and (46) for the case in which $\theta = 1.5\,\pi$ are shown in Table 3, together with the values read from the author's curves in Fig. 4.

L. J. Mensch,[32] M. Am. Soc. C. E. (by letter).[32a]—Among the great number of similar problems occurring in the construction of tanks, grain elevators, ships and submarines, airplanes, arched dams, and beam and flat-slab foundations on yielding soil, the author's paper treats of the simplest case. By no means is it simple to the every-day engineer; very likely the author has spent years in developing the analysis and in computing the data on which Figs. 2, 3, and 4 are based.

This particular problem was solved in connection with the design of high-pressure boilers in 1878.[33] With the introduction of reinforced concrete, many papers appeared on secondary stresses in tanks, domes, and foundations.[34] After 1904 the Society published the most important analytical and experimental investigations ever made on arched dams, which is a vastly more difficult problem.

American designers will look with awe at the forbidding appearance of Equations (14) and will hesitate to use them because of possible typographical and other errors. Other authors have developed short-cuts and have constructed tables and diagrams.[35] Evidently, the author took cognizance only of the short-cuts published in American literature on the design of circular tanks with a characteristic, $\frac{h^2}{r\,t}$, greater than 10, and, for his own practice, developed a design method for tanks with lower characteristics.

How much confidence do practical engineers and owners have in such involved theories? They have very little, at least in the writer's experience. In all likelihood, the chief engineer or owner will direct the designer to use the same ring steel as if the wall were unrestrained at the bottom and, for greater safety, to use the vertical reinforcement required by this analysis.

Such practice prevailed 30 yr ago (1909) in the design and construction of arched bridges. Laboriously, by the elastic theory, using the summation

[32] Civ. Engr. and Constructor, Chicago, Ill.

[32a] Received by the Secretary May 1, 1939.

[33] "Theorie der Elasticitaet und Festigkeit," by F. Grashof, Berlin, Germany, 1878, pp. 317–329.

[34] *Handbook on Reinforced Concrete*, by F. Emperger, Berlin, Germany, 1907, and succeeding editions; Volume Fluessigkeitsbehaelter and other volumes.

[35] *Loc. cit.*, Third Edition, Vol. 5, 1923, pp. 87–112.

method, designers determined the moments acting at critical points. They did not use these moments for the design of the arch, but drew the line of pressure for live loads on half of the span after many trials, and made the arch section deep enough so that the line of pressure did not fall outside of the middle third at any point. Many concrete bridges were built at that time which were heavier than stone bridges built many hundreds of years ago, and are still in use. When it became fashionable to use reinforced concrete, an ample supply of patented bars were "thrown in for good measure."

The same conditions may be found even to-day with respect to rigid-frame bridges. Trade associations scatter pamphlets by the ten thousands, praising the summation method. The final result is that such bridges have been built, according to the writer's opinion, for a moment of $\frac{W\,L}{8}$ at the haunches so that nothing serious could happen if the bridge cracked at the center; and sufficient concrete and steel were used in the center so that the bridge could not fail if the haunches or abutments were insufficiently designed.

The author's analysis is based on the assumption that the circular wall is fixed at the bottom, both for rotation and horizontal and vertical movement. This condition is obtainable only in the rare case in which the wall is anchored in a trench of solid rock; in most designs which came to the writer's notice, the designer tried to prevent rotation by a special wide footing beneath the wall and floor. Sometimes the cost of the footing was greater than the cost of the wall. In higher tanks the horizontal force from the water pressure at the bottom of the wall was sufficient to move the wall $\frac{1}{4}$ in., $\frac{1}{2}$ in., and in some known cases 1 in. and more, playing havoc with the ordinarily poorly designed joint between wall and floor, and this practice resulted in serious leakage. The ring stresses become greatly increased due to such movements. If, by some miracle, the wall does not move, it is a practical certainty that there will be a rotation of the wall at the bottom, especially if the footing was skimped in order to save money. Such a rotation will diminish the moment at the base, will increase the positive moment more rapidly, and will also increase ring stresses.

Where a group of tanks is to be built, it is uneconomical to use circular tanks, and covered square tanks may be less costly than open round tanks built separately. In such tanks the walls are computed for top and bottom restraint, and the corner restraints are determined by an analysis similar to that presented by the author.

Domes of large span, covering circular tanks, will exert a large thrust at the top of the wall, resulting in a horizontal movement even at that point, where a heavy ring girder is used to take up the thrust, thereby increasing the ring stresses and causing secondary bending in the walls.

JENS EGEDE NIELSEN,[36] ASSOC. M. AM. SOC. C. E. (by letter).[36a]—The author has revived the problem of a cylindrical shell, closed at one end, in its simplest and most practical form where the thickness of the shell is uniform.

[36] Civ. Engr., C. B. & Q. R. R., Chicago, Ill.

[36a] Received by the Secretary April 29, 1939.

The general problem was treated in detail by Professor Grashof [37] in 1874. However, his solution of the fourth-order differential equation is not directly applicable to practical use, and the author, therefore, is to be congratulated for offering a solution which embraces most considerations required by the designing engineer.

In order to render "unto Caesar the things which are Caesar's," it is only fair to state that it is an erroneous conclusion of the author that all concrete tanks constructed previous to the appearance of this paper had been designed haphazardly or inefficiently. It has long been known that the shape of the elastic curve is clearly a function of $\frac{h^2}{r\,t}$ as the author correctly states. Based on this fact, it has been possible to make a close approximation of the distribution of the load between the ring and the cantilever, and tables compiled on this basis may be found in offices where concrete water tanks are designed. These tables are probably as efficient as the author's diagrams, and naturally should be so, as they are fundamentally alike.

In water tanks it is important that no cracks develop; otherwise, due to the high internal pressure, water will penetrate and in time destroy the reinforcement and the wall. Experiments at the University of Illinois indicate that a stress of 6 000 lb per sq in. should not be exceeded in order to eliminate destructive hair cracks in the concrete. The author recommends 12 000 lb per sq in. as the proper working stress. Here, then, is a difference of opinion, and designers are again confronted with a method of design more refined than is warranted by other considerations.

If insufficient additional reinforcement is provided to take care of volumetric and temperature changes, as suggested by C. P. Vetter, Assoc. M. Am. Soc. C. E.,[38] and a stress in excess of 6 000 lb per sq in. is permitted, fine hair cracks will develop and, in time, the ring steel will be required to sustain all tension due to continuing diminished adhesion. The modulus of elasticity E_0, therefore, will not be equal to E when this stage has been reached, but will probably be nearer ten times E. Hence, the value of β and θ must be modified and the diagrams adjusted accordingly. In spite of the fact that this is one of the extremely rare cases in engineering where the exact live load is known, designers are confronted with a disturbing uncertainty which, when they are pressed for time, they overcome by "playing safe," providing for the most extreme possibility.

It may be opportune here to draw the attention to the importance of continuous operation when placing concrete in the forms. If the operation is interrupted, the finished tank will develop a horizontal leak precisely at that level, and it requires considerable care and expense to correct it afterward. The writer recalls one tank for which the cost was increased several hundred dollars on account of a short lunch period.

This paper is highly meritorious in that it provides an avenue of approach which may be extended at leisure to conform to the individual designer's ideas.

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[37] "Theory of Elasticity," by Franz Grashof, Berlin, Germany, 1874.

[38] *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1039.

LLOYD S. DYSLAND,[39] JUN. AM. SOC. C. E. (by letter).[39a]—This paper provides a distinct advance in accuracy and ease of design of an indeterminate problem of frequent occurrence, particularly by the inclusion of Fig. 4. Unfortunately the plotting of C_{-m} is not consistent with such accuracy. However, it can be calculated from the expression $M = \frac{w\,h^3}{2\,\theta^3}(2\,C_1 + 2\,C_2 + \theta - 1)$, which is derived from Equation (14*a*), by using a value of $\theta = \pi\,h\sqrt{\frac{0.176}{r\,t}}$ and the values of C_1 and C_2 in Fig. 2. A direct means of computing the height of the point of inflection of the wall moment is quite desirable but is not included.

The analysis presented provides for an inside fluid-pressure loading for the entire wall height and may, by a change of sign, be applied to exterior fluid-pressure loads for the entire wall height. However, it cannot be indiscriminately used for loading to any fraction of the wall height, although the author's statement following Table 1 would lead the reader to that conclusion.

From physical considerations it seems that with partial loading, when use of the loaded height gives values of θ less than about $1.0\,\pi$, the mathematical analysis may be seriously misleading. Fig. 4 shows that for values of θ less than $1.0\,\pi$ the "cantilever" effect (the effect more nearly resembles that of a beam fixed at one end and supported at the other than it does a cantilever) induces a considerable circumferential stress at the top of the loaded portion of the wall which is supposed to be the top of the tank. The analysis gives no stress where there is no load; therefore the wall above the loaded height would have no stress. Thus, for shallow tanks, the analysis would indicate that a highly stressed ring of wall existed adjacent to one which carried no load at all. Such a condition is unreasonable.

Actually, when the wall extends some distance above the height of fluid level, the upper part of the wall must provide (in shallow tanks) additional top restraint to the vertical staves. The result will be a redistribution of hoop stress and a decrease in moment at the base.

Manning,[40] in an analysis based on the same fundamental equation (Equation (10*b*)), provides charts to give exact values of moment and shear at the base. However, he uses approximations to determine hoop reinforcement in deep tanks and gives no solution for finding hoop reinforcement in shallow tanks. Leeper[41] greatly simplifies his solution of the differential Equation (10*b*) by neglecting certain terms, and by further approximations derives simple algebraic expressions for the design factors.

For the height above the base at which the point of inflection of wall moment occurs, Manning[40] suggests $0.61\,\sqrt{r\,t}$ and Leeper[41] gives $0.56\,\sqrt{r\,t}$.

It frequently happens that greater economy can be realized by using a wall cross-section which tapers toward the top. Some conception of the effect of

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[39] Designer, Consoer, Townsend & Quinlan, Chicago, Ill.

[39a] Received by the Secretary May 15, 1939.

[40] "Reinforced Concrete Design," by G. P. Manning, Second Edition, p. 331.

[41] "Design of Cylindrical Tanks," by L. D. Leeper, Jun. Am. Soc. C. E., *Civil Engineering*, Vol. 3, No. 11, November, 1933, pp. 598–600.

tapering walls can be gathered from an article by H. Carpenter[42] in which he gives load-distribution and base-moment curves for tanks with walls of triangular cross-section and of rectangular cross-section for several shallowness ratios.

To determine the relative economy of fixed-base design, compared with hoop-tension design, a careful estimate was made recently on two 75-ft diameter sewage digestion tanks, 24 ft deep, in which the chief difference (in addition to the redistribution of reinforcement) was a graphite painted copper joint at the base of the wall designed for hoop tension only. The estimate was \$2 000 lower on the fixed-base tank.

Basil Sourochnikoff,[43] Esq. (by letter).[43a]—In Mr. Salter's numerical example only stresses due to hydrostatic pressure and curing are computed. As the structural stresses are low, the temperature stresses may be expected to be proportionally important. Assume, for instance, that, due to a decrease of temperature, the rings are shortened. The base, being restrained by the foundation, may not undergo the same shortening. For an effective difference of 10° F between the average temperature of the walls and that of the base, a ring, if not restrained by the cantilever, would have the radius decreased by $0.000006 \times 10 \times 35 \times 12 = 0.025$ in. Due to the hydrostatic pressure alone, a ring at mid-height of the tank, if not restrained by the cantilever, would have its radius increased by $\frac{62 \times 6.28 \times 35}{12 \times 9} \times \frac{1}{2\,500\,000} \times 35 \times 12 = 0.021$ in. Therefore, it may be expected that the temperature stress in the cantilever is of the same order of magnitude as the hydrostatic stress.

The stresses due to a difference of temperature between the walls assumed constant throughout, and the base, may be analyzed by the same method as the hydrostatic stresses are analyzed in the paper. For a temperature difference of $T°$ F, the rings, if not restrained by the base, would have the radius decreased by $y_0 = 0.00006\, T \times r$ ft. In this deformation they are restrained by the base. It is seen that the differential equations and their integration are exactly the same as for hydrostatic pressure, with $w = 0$. For the determination of arbitrary constants one may assume at the top that $\frac{d^2y}{dx^2} = 0$; and $\frac{d^3y}{dx^3} = 0$; and at the bottom $y = -\, y_0$, and a certain arbitrary angular deformation $\alpha_0 = \frac{dy}{dx}$.

The stress distribution analyzed by Mr. Salter's method is certainly more exact than that obtained by other approximate methods. However, it seems that it can be extended with advantage to take into account the secondary stresses, which may be important.

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[42] "Calculation of Cylindrical Tanks with Rectangular, Triangular, or Trapezoidal Wall Sections," by H. Carpenter, *Concrete and Construction Engineering*, Vol. 24, No. 6, June, 1929, pp. 345–353.

[43] Structural Engr., St. Paul, Minn.

[43a] Received by the Secretary May 15, 1939.

REGIS F. FEY,[44] JUN. AM. SOC. C. E. (by letter).[44a]—Progress in circular tank design is aided materially by Mr. Salter's paper. The effect of shell restraint or fixation at the base (a condition which tank designers realized existed, yet hesitated to consider in their designs) has been analyzed very clearly in this paper.

The importance of the type and degree of restraint of the tank wall at the base is somewhat difficult to determine. For a rational design it is reasonable to discount any appreciable horizontal or vertical movement at this point, but it is more difficult to conclude upon a reasonable assumption with respect to rotation about this point. The combined effect of the wall ring tension and the bottom restraint tends to cause a rotation. If the bottom is rigid enough to counteract any rotation, the cantilever effect of the vertical wall segments will come into full effect in carrying their share of the horizontal load; but if the bottom should deflect somewhat, due to the effect of rotation, the load-carrying capacity of the cantilever wall segments will be reduced somewhat. Therefore, considering the wall rigidly attached to the bottom, the degree of restraint may be considered as a function of the rigidity of the bottom.

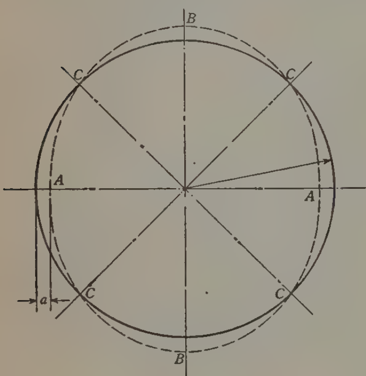

FIG. 8

When investigating a thin circular wall for external pressures, the resultant effects upon the wall are different from those caused by internal pressures. The stresses in the wall may be treated from the standpoint of the flexural rigidity of a ring. For stress investigation a nominal deformation from the true circle should be considered.

The deformation may be either elliptical or in the form of so-called "flat spots" in the wall (Fig. 8). For an elliptical deformation the maximum bending moment is at Point A. Since the ratio of the applied external pressure is small with respect to the critical external pressure at which the ring would collapse (that is, if no further ellipticity would result from the applied external pressure), the moment at Point A is

$$M_A = h\,q\,a\left(r + \frac{a}{2}\right) \qquad \text{(47a)}$$

in which, in addition to the notation of the paper: q = the applied pressure; and, a = radial displacement = one-half "out-of-roundness"; that is, one-half the difference between the diameter of the true circle and the minimum diameter

[44] Engr., Pittsburgh Des Moines Steel Co., Pittsburgh, Pa.

[44a] Received by the Secretary May 15, 1939.

of the irregular section. The moment at Point B is in the opposite direction:

$$M_B = h\,q\,a\left(r - \frac{a}{2}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(47b)$$

Since a is small compared with r, the moments at Points A and B may be considered equal for all practical purposes, or:

$$M_A = M_B = h\,q\,a\,r\ldots\ldots\ldots\ldots\ldots\ldots\ldots(48)$$

The moments at Points C are zero.

So called "flat spots" in the circular wall are not plane surfaces but sections which have a lesser degree of curvature. For moment computations a section of wall including, and adjacent to, a flat spot may be considered elliptical in shape. Treating the section as before, the maximum moment at the center of the flat spot will be that given by Equation (48). In this case, however, the radial displacement, a, is equal to the "out-of-roundness." The designer should determine whether care will be taken to prevent any localized irregularities in building the wall section. If this cannot be ascertained, the moment due to a flat spot should be considered; otherwise, the moment due to a general ellipticity should be used. From several measurements, the writer has found that an "out-of-roundness" of 0.25% of the diameter may be anticipated in a circular concrete tank, considering average construction methods. These moments in the wall should be considered in conjunction with the compression in the wall caused by external forces acting on a ring considered circular in shape. These factors give a condition analogous to an eccentrically loaded column.

This treatment of external pressures on walls of circular tanks does not take into consideration the restraint of the bottom, the cantilever effect of the vertical wall segments, or any resistance to torque imposed by the moments due to irregularities, which are of minor importance except in very shallow tanks of large diameters.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

THEORY OF LIMIT DESIGN

Discussion

By Messrs. Joseph A. Wise, Alfred M. Freudenthal, Hans H. Bleich, Alfred S. Niles, and A. Floris

Joseph A. Wise,[35] M. Am. Soc. C. E. (by letter).[35a]—The philosophy upon which the theory of limit design is based appears to be a sound one, but before it can be accepted as a basis for the design of structures, it will require much clarification and perhaps amplification. Of the three dicta proposed, the first one is vague and fails to define a statically indeterminate structure. In one sense, all members, reactions, and restraints function to the same end—that of resisting the applied forces without failing or deforming excessively. The essential characteristic of a statically indeterminate structure is that it has redundant members, reactions, or restraints in excess of the number just sufficient to maintain static equilibrium. The third dictum is a proposition that is not correct. The tower shown in Fig. 15 is at least doubly indeterminate if Member $E\,H$ is considered not acting; yet the failure of two members, such as $E\,H$ and $F\,H$ or $C\,F$ and $D\,F$, would suffice to cause complete collapse instead of the three required by the third dictum. If it should be argued that, with reference to any one bay, this structure is singly indeterminate only, and therefore the failure of two members in one bay constitutes the failure of $n + 1$ members, one need only point out that Members $C\,D$ and $E\,F$ could be chosen as redundants. Thus, it would become necessary to define "bay" so as to include only one of these two members. For more complex structures any such definition would be difficult, if not impossible. As a matter of fact, the third dictum obscures, rather than clarifies, the manner in which structures may fail. In its place, the following is proposed:

A framed structure will collapse completely when a sufficient number of members have buckled or yielded, so that the remaining part of the structure is statically unstable. The "remaining part" of the structure is defined as that

Note.—This paper by J. A. Van den Broek, M. Am. Soc. C. E., was published in February, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1939, by Messrs. John H. Meursinge, I. K. Silverman, Edward Godfrey, Basil Sourochnikoff, E. Mirabelli, C. M. Goodrich, George Winter, and Francis E. Simpson.

[35] Associate Prof., Civ. Eng., Univ. of Minnesota, Minneapolis, Minn.

[35a] Received by the Secretary April 14, 1939.

part formed by omitting all members that cannot act to resist any increase in load, because they have buckled or yielded, but including any forces which they supply. It must be noted that even after a member has buckled, due to a compressive force, it may still be available to resist tensile forces, and therefore, whenever the increase in load tends to produce tension in a buckled member, that member should not be omitted in forming the remaining part of the structure.

In the paper, the tacit assumption is made that after a member has buckled it will continue to have the same axial stress until the structure collapses. This is not entirely correct. Fig. 27 shows a pin-connected member subjected to an axial load P. It will be assumed that the member is initially straight, and that the axial load is applied so that the member does not bend until the critical or buckling load is reached. If, now, the distance between the ends is decreased by a distance ΔL, the load P will not change as long as the member remains elastic; that is, no part of it is stressed beyond the yield point. The member will bend, however; the work done by the load will be stored as elastic strain energy of bending. Assuming that the elastic curve is a sine curve

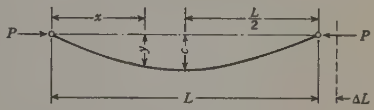

FIG. 27

$$y = c \sin \frac{\pi x}{L} \quad \text{(14)}$$

a close approximation for ΔL can be obtained as follows:

$$\Delta L = \tfrac{1}{2} \int \left(\frac{dy}{dx}\right)^2 dx = \frac{c^2 \pi^2}{4 L} \quad \text{(15)}$$

and

$$c = \frac{2}{\pi} \sqrt{L \, \Delta L} \quad \text{(16)}$$

The moment at the center will then be

$$M = P c \quad \text{(17)}$$

As long as the material is not stressed beyond the yield point, the axial load will not change appreciably. The small change in axial load, which can be determined by using the exact expression for radius of curvature instead of the approximate value, $\dfrac{1}{\frac{d^2y}{dx^2}}$, can be neglected in this case.[36] When the member becomes stressed beyond the yield point at the center, however, the moment there cannot be appreciably increased unless the axial load P decreases. As fibers closer to the neutral axis than the extreme fiber reach the yield point, the resisting moment will increase slightly; but it will be sufficiently accurate for this analysis to assume that the moment, M', which exists when the extreme

[36] "Die Knickfestigkeit," by Rudolf Mayer, Section 4.

fiber reaches the yield point is the limiting bending moment. Thereafter the axial load will decrease as ΔL and c increase, since

$$P = \frac{M'}{c} \qquad (18)$$

Furthermore, as P decreases, the axial length of the member will tend to increase, and this will tend to increase c still further. Calling ΔP the decrease in P, and Δc the increase in c, Equation (18) yields:

$$(P - \Delta P) = \frac{M'}{c + \Delta c} \qquad (19)$$

and including the axial change in length,

$$\Delta L = \frac{(c + \Delta c)^2 \pi^2}{4 L} - \frac{\Delta P\, L}{A\, E} \qquad (20)$$

Substituting the value of $c + \Delta c$ from Equation (19) in Equation (20):

$$\Delta L = \frac{(M')^2 \pi^2}{4 L (P - \Delta P)^2} - \frac{\Delta P\, L}{A\, E} \qquad (21)$$

Furthermore, as P decreases, the unit axial stress due to it will decrease and thus the section will be able to resist a greater moment when the extreme fiber stress is at the yield point. Calling s the unit stress, in extreme fiber due to moment M', the increased resisting moment will be $M' \left(1 + \frac{\Delta P}{s\, A}\right)$ and Equation (21) becomes:

$$\Delta L = \frac{(M')^2 \pi^2 \left(1 + \frac{\Delta P}{s\, A}\right)^2}{4 L (P - \Delta P)^2} - \frac{\Delta P\, L}{A\, E} \qquad (22)$$

in which A is the area of member. Equation (22) is a cubic, ΔP being the unknown quantity to be determined. It can be solved easily by trial when numerical values are given. Although the axial deformations may be of the order of elastic deformations, the lateral deflections will be much larger. For example, a 3-in. by 3-in. by $\frac{1}{4}$-in. angle, 8 ft long, will have a buckling load (as a pin-ended column) of P = 16 100 lb. It will shorten under this compression, $\frac{16\,100 \times 96}{1.44 \times 30 \times 10^6} = 0.0358$ in. If the ends are caused to move toward each other 0.01 in., the lateral deflection, c, will be $\frac{2}{\pi} \sqrt{96 \times 0.01} = 0.623$ in., or more than 60 times the axial change in length. The moment at the center will be $16\,100 \times 0.623 = 10\,040$ in.-lb. The extreme fiber stress due to this moment will be $\frac{10\,040 \times 1.19}{0.50} = 23\,900$ lb per sq in. at the heel. This stress, combined with the axial compressive stress of $\frac{16\,100}{1.44} = 11\,200$ lb per sq in., would produce a stress of 35 100 lb per sq in., a value close to the yield point of standard structural steel. Any further displacement of the ends of the mem-

ber toward each other would be accompanied by a rapid increase in lateral deflection and a decrease in axial load. Thus a further decrease in length of 0.01 in. would make $\Delta L = 0.02$ in. Equation (22) becomes,

$$0.02 = \frac{10\,040^2 \times \pi^2 \cdot \left(1 + \frac{0.42 \times \Delta P}{1.44 \times 23\,900}\right)^2}{4 \times 96\,(16\,100 - \Delta P)^2} - \frac{\Delta P \times 96}{1.44 \times 30 \times 10^6}$$

By trial it will be found that $\Delta P = 5\,700$ lb and

$$c + \Delta c = \frac{M'\left(1 + \frac{\Delta P}{A\,s}\right)}{P - \Delta P} = \frac{11\,700}{10\,400} = 1.125 \text{ in.};$$

and $\Delta c = 1.125 - 0.623 = 0.502$ in. Thus, it is seen that, even for axial movements of the order of elastic deformations, the lateral deflections become very large, and the axial load decreases rapidly.

A comparatively simple structure (Fig. 28) will be used to illustrate these principles. Beginning with a load, P, of 200 kips, the stresses in the members, determined by the method of consistent deflections, are shown in Table 1(a),

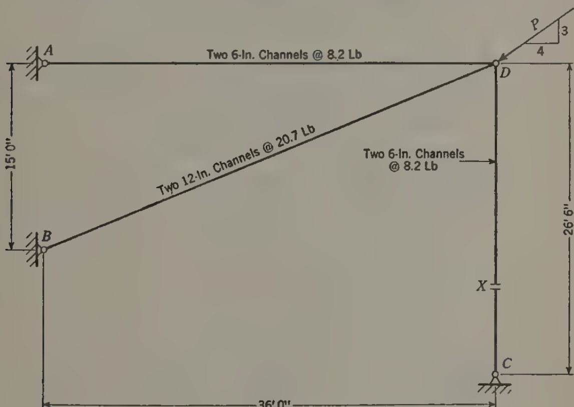

FIG. 28

the final step being $X = -\dfrac{4\,940}{70.8} = -69.9$. The members are assumed to be built so that their slenderness ratios, $\dfrac{L}{r}$, are at least as small transversely as they are in the plane of the structure. Column (10), Table 1, gives the buckling stress, computed by Euler's formula. If the load is increased to 210 kips, Member $A\,D$ will buckle. At this load the stresses will be those shown in Fig. 29(a). The remaining part of the structure consists of Members $B\,D$ and $C\,D$ only. If a load $P = 1$ lb is applied, the stresses are as shown in Fig. 29(c). Member $C\,D$ has a margin of strength of $76.5 - 73.4 = 3.1$ kips before it

TABLE 1.—Illustrative Example

Member (see Fig. 28)	(a) Computation of Stresses by Consistent Deflections									(b) Computation, Horizontal Movement of Point D			
	Length, L, in feet	Area, A, in square inches	$\frac{L}{A}$	Stress, S', in kips per square inch	u	$S'u\frac{L}{A}$	$u^2\frac{L}{A}$	Stress, S, in kips per square inch	Buckling stress, in kips per square inch	Stress, S, in kips per square inch	$\frac{L}{A}$	u	$Su\frac{L}{A}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
AD	36.0	4.78	7.53	+128	+2.40	+2 310	+43.4	− 39.5	− 41.5	....	...	....	...
BD	39.0	12.03	3.24	−312	−2.60	+2 630	+21.9	−130.5	−346.0	−10.1	3.24	−1.083	+35.4
CD	26.5	4.78	5.54	0	+1.00	0	+ 5.5	− 69.9	− 76.5	− 3.1	5.54	+0.417	− 7.2
Total	....	....	....	....	....	+4 940	+70.8	...	....	....	....	....	+28.2

buckles. Thus, if a load of $\frac{3.1}{2.67} = 11.6$ kips is applied at Point *D*, Member *C D* will just buckle, and the added stresses will be as shown in Fig. 29(d). However, this is on the supposition that Member *A D* continues to exert a force of − 41.5 kips. It is necessary to find the horizontal movement of Point *D*. This is done in Table 1(b), which indicates that *D* moves $\frac{28.2 \times 12}{30 \times 10^3} = 0.01128$ in. toward the left. Applying Equation (16), $c = \frac{2}{\pi}\sqrt{432 \times 0.01128} = 1.41$

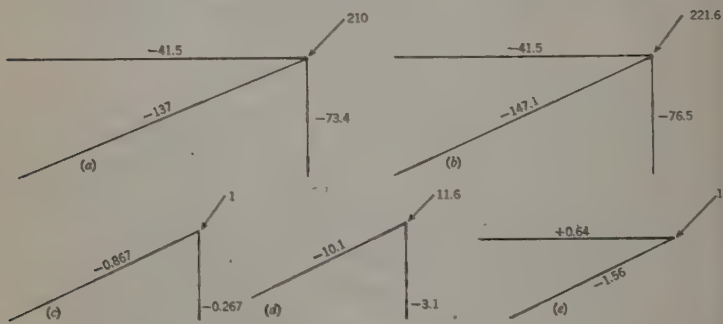

Fig. 29

in. The moment at the center would be $41.5 \times 1.41 = 58.5$ in-kips. The extreme fiber stress due to this bending moment would be $\frac{58\,500}{8.6} = 6\,800$ lb per sq in.

The axial stress would produce $\frac{41\,500}{4.78} = 8\,680$ lb per sq in., or the combined stress would be 15 480 lb per sq in., which is well within the yield point. The total stresses for the frame at this stage are shown in Fig. 29(b).

The next step will be to consider Members $A\,D$ and $B\,D$ as the remaining structure after Member $C\,D$ buckles. Member $A\,D$ is included since an increase in load will tend to induce tension in that member. Fig. 29(e) shows the stresses due to $P = 1$ lb. The very smallest increase in load will permit Member $A\,D$ to transfer the elastic strain energy stored in it, due to the previous movement at Point D of 0.01128 in., into Member $C\,D$. Since no appreciable change in load or stress is assumed to occur during this transfer, Member $B\,D$ will merely rotate about Point B, guiding Point D along a line at right angles to Member $B\,D$. Therefore Point D moves vertically a distance equal to $\frac{12}{5} \times 0.01128 = 0.0271$ in. Again applying Equation (16):

$$c = \frac{2}{\pi} \sqrt{318 \times 0.0271} = 1.87 \text{ in.};$$

and, the moment at the center would be $76\,500 \times 1.87 = 143\,000$ in-lb. The maximum stress due to combined bending and axial stress would be $\frac{143\,000}{8.6} + \frac{76\,500}{4.78} = 32\,630$ lb per sq in., compression. Assuming a yield point of 33 000 lb per sq in., Member $B\,D$ has a margin of strength of 198.9 kips left at this stage. It would require an added load of $\frac{198.9}{1.56} = 127.6$ kips to produce this stress. However, since Member $C\,D$ has practically reached its yield point, the load in it will now tend to decrease as Point D moves downward, and therefore the stress in Member $B\,D$ will be increased by this effect. Thus, it is obvious that the structure will not sustain an added load of 127.6 kips.

Proceeding by trial, first add a load of 10 kips; the added stresses will be $+ 6.4$ in Member $A\,D$ and $- 15.6$ in Member $B\,D$, assuming no unloading from Member $C\,D$. The vertical deflection of Point D will be 0.1149 in. ($= \Delta L$). For this case, ΔP will be 41 500 lb, the stress in Member $A\,D$ is changed from $+ 99.5$ to $+ 58.0$, and the stress in Member $B\,D$ is changed to $- 254.9$ kips. This change, in turn, will cause Point D to deflect further, and the cycle will be repeated. It will be found that only about 5 kips can be added before collapse occurs; hence the limit load for this structure is about 226.6 kips. Thus, the structure will carry more than the limit load as determined from Members $A\,D$ and $C\,D$; and, also all three members buckle before final collapse occurs.

If a similar, step-by-step analysis is applied to the tower in Fig. 15, it will be found that Member $C\,F$ will buckle first, when the load at the top of the tower reaches 33.5 kips. It will be found that an additional load of 8 kips at the top will then cause the buckling of Member $D\,F$, and therefore the structure will collapse at a total load of 41.5 kips, instead of the 60 kips for which it was designed. Thus, it can be seen that the procedure proposed by the author is not valid unless checked by a step-by-step analysis of the behavior of the structure as a whole; and, that it is not correct to assume that a member will continue to have the same stress after it has buckled as it had at the buckling point.

One other phase of the subject should be emphasized. The dead load of a structure is not ordinarily likely to be increased appreciably, and only the increase in live load can lead to a failure due to over-load. By present methods

of design, structures with large ratios of dead load to live load, such as rein-forced concrete structures, are penalized in comparison with others (such as stee structures). Under limit design, concrete structures would probably have pro-portionately greater reductions in sizes of members than would steel structures

There is also an inconsistency in the basic paper, in that the author state that, in the theory of limit design, the emphasis is to be shifted from "stress' to "permissible deformation." Nevertheless, the paper proceeds to discuss th stresses chiefly, a procedure which it had previously condemned. As a matte of fact it does not matter whether one uses the limiting deformation or the stres at which it occurs, as the criterion for the strength of a member, except i cases where two-dimensional or three-dimensional states of stress occur. Ther the deformations depend on all stresses and not on the stress in the direction o the deformation alone. Therefore, the subject should be separated into tw parts: (1) Structures with uni-axial states of stress; and, (2) structures wit bi-axial or tri-axial states of stress. For the first type this discussion is perti-nent; for the second type, the question of the controlling factor at failure wil depend upon the material; for brittle materials generally, the deformations wil control; and, for ductile materials, a shearing stress which is also a function o the maximum tensile stress, as indicated by the Guest-Hancock, or Moh theory of failure, will control.

ALFRED M. FREUDENTHAL,[37] ASSOC. M. AM. SOC. C. E. (by letter).[37a]—Ver ably the author has presented a picture of the behavior of steel structure stressed above the yield limit. A comprehensive treatment of the theory o limit design based on the ductility of steel is destined to meet with seriou difficulties as, at the present time, the main features of the theory of plasticit are in a state of flux. Under the stipulated conditions and the assumption made, the author's point of view may be justified. However, the write believes that these conditions are encountered in only a restricted number o cases in practice and that, in general, the assumptions are not valid. Further-more, a number of important influences have been insufficiently considere (or not at all); and this might involve fundamental changes in the conceptio of the theory of limit design.

(1) The author asserts that the capacity load of an n-time staticall indeterminate structure is reached after $(n + 1)$ members have all reache their elastic or buckling limit strength. With regard to the danger of buckling this statement, in its general form, is quite a dangerous one because the author' conviction that "in the consideration of compression members the logic i essentially the same as that applied to tension members" does not seem to b based on evidence. Furthermore, the author is not at liberty to choose buckling formula. The introduction of Euler's formula is not subject t "preference"; its validity is purely a question of whether buckling occur before the yield point is reached or after it has been exceeded.

[37] Res. Engr., Marine Trust, Tel-Aviv Port, Tel-Aviv, Palestine; Lecturer on Bridge Engineering, Hebrew Institute of Technology, Haifa, Palestine.

[37a] Received by the Secretary April 17, 1939.

As the author states, the load-deformation relationship after the buckling has begun is of particular importance to the entire problem. This relationship (see Fig. 30) differs essentially in the cases of elastic and plastic buckling.[38]

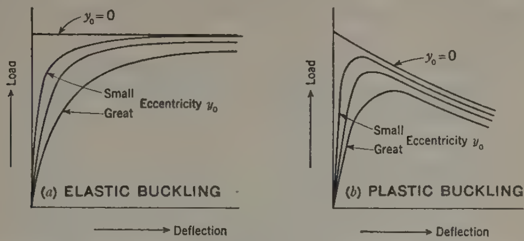

FIG. 30.—ELASTIC AND PLASTIC BUCKLING; LOAD-DEFLECTION CURVE FOR VARIOUS ECCENTRICITIES

Thus, the behavior of statically indeterminate structures is materially affected by the type of buckling involved.

To make this fact clear, the behavior of the structure in Fig. 31 may be considered. With an increasing load, P, the capacity load of the structure is reached in different ways, depending on the proportion of stresses in, and the buckling stiffness of, the struts and the vertical bar. According to the theory of elasticity the force acting in the vertical bar has the value

$$X = \frac{P}{1 + 2\,\frac{A_s}{A_v}\sin^3\phi} \quad \text{(23)}$$

in which A_v and A_s, respectively, are the cross-section areas of the members indicated. For $\phi = 30°$ and $A_s = A_v$, the force in the vertical bar is $X_v = 0.8\,P$; and the force in the struts, $X_s = 0.2\,P$. Because of this difference between stresses in the elastic state, below the stability limit, the members of the structure cannot be stressed to capacity simultaneously. The following are the different possibilities of reaching the capacity load of the structure:

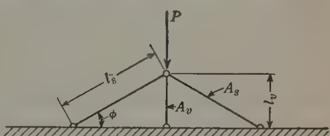

FIG. 31.—A STATICALLY INDETERMINATE SYSTEM

(a) The buckling limit of the vertical bar may be exceeded before the stress $X_s = \frac{X_v}{A_v}$ reaches the yield point. Under increasing lateral buckling deflection of the vertical bar and almost constant buckling resistance (see Fig. 30(a)), the forces in the struts increase gradually, until their elastic buckling limit (and, with it, the capacity load of the entire structure) is reached. Hence, the structure collapses under stresses which are, in all members, below the yield point.

(b) The buckling limit of the vertical bar may be reached after the stress has exceeded the yield point. Referring to Fig. 30(b), the resistance of the

[38] "Plasticity," by A. Nadai, McGraw-Hill Book Co., Inc., New York, N. Y., 1931.

bar decreases considerably with increasing lateral deflection, thus involving a rapid increase of the forces acting in the struts. Subsequently, their buckling limit is reached and the structure collapses. If the buckling resistance of struts and vertical bar does not differ essentially, the actual capacity load of the structure exceeds only slightly the capacity load based on the buckling limit of the bar itself. Only if the stiffness of the struts has a far greater value than that of the vertical bar may the actual capacity load of the structure exceed, greatly, its apparent capacity load, based on the buckling limit of the vertical bar itself.

To illustrate the rapid decrease of buckling resistance in a bar,[39] the stress of which has exceeded the yield point, Fig. 32 gives the results of tests on

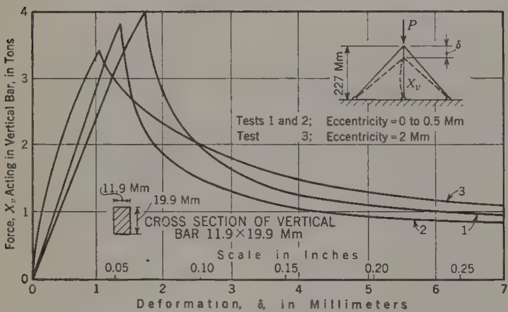

FIG. 32.—RELATION BETWEEN FORCE AND DEFORMATION FOR VARIOUS DEGREES OF ECCENTRICITY.

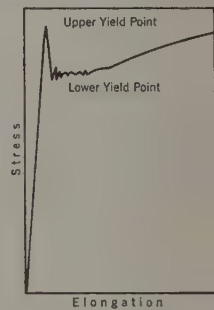

FIG. 33.—STRESS ELONGATION CURVE OF STEEL IN TENSION

redundant steel bars with a yield point of 33 500 lb per sq in. The relationship of load and the decrease of the vertical distance between the end points of the deformed bar[39] are shown.

It may be concluded from the foregoing that the behavior of a redundant compression member of a statically indeterminate structure, stressed to capacity under continually increasing load, differs essentially from the behavior of a tension member, under similar conditions. The "fear of buckling" which, as the author remarks, "appears deeply ingrained" is fully justified not only from the point of view of the theory of elasticity but even more from the point of view of the theory of (plastic) limit design. Hence, Bleich's statement[6] concerning compression members is correct: The theory of limit design must never be applied to trusses. The statement by Professor Kist,[13] however,

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[39] "Three Contributions on the Loading Question of Statically Indeterminate Steel Trusses," by E. Chwalla, Publications of the International Assoc. for Bridge and Structural Engineering, Zurich, Switzerland, Vol. 2, 1933–1934.

[6] "Über die Bemessung Statisch unbestimmter Stahltragwerke unter Berücksichtigung des Elastisch-Plastischen Verhaltens des Baustoffes," by Hans Bleich, *Bauingenieur,* 1932, Heft 19/20, p. 261.

[13] "Does a Theory of Strength, which Is Based on the Assumption of Proportionality between Stress and Strain, Lead to Good Construction of Steel Bridges and Buildings?" by N. C. Kist, Inaugural address, Technical University, Delft, Holland, 1917. Also, "Ductility as a Base for Design—Computation of Steel Bridges and Structures instead of Proportionality of Stress and Strain," by N. C. Kist, International Congress for Metallic Structures, Liége, Belgium, 1930.

that "the engineer need not apply the theory of elasticity in order to assign values for statically indeterminate quantities" is exaggerated: it applies only to a very limited number of structures; and, in its general form, it is open to strong criticism.

Until now one fact has not been considered. In practice, compression members do not consist of bars of rectangular sections, but of angles, channels, and joists. The stability limit of such sections is generally not reached gradually as assumed theoretically for rectangular sections. The particular form of the section almost always causes elastic or plastic buckling, with simultaneous torsion, bending, or premature buckling of the flanges, and with an almost instantaneous, abrupt loss of the buckling resistance. This phenomenon leads to a load-deformation relationship of plastic buckling which is far more unfavorable with respect to the influence of the resistance of the compressed member on the capacity load of the structure than the load-deformation relationship assumed in the theory of limit design.

(2) The behavior of compression members in trusses, as assumed in the paper, is only one of the features that makes the general conclusions advanced by the author open to criticism. It will be shown that the assumption underlying the theory of limit design for members acting under bending stresses cannot be fully endorsed, either.

The assumption of an ideal plastic behavior of the material, according to Fig. 2, has proved to be valid only for special kinds of steel and under particular forms of loading. During the past few years the foregoing assumption has been strongly opposed by various writers. Their objections are based on a considerable number of tests which have shown that the existence of an upper yield point (Fig. 33), as well as the apparent rise of the yield stress, due to non-uniform distribution of stresses over the sections (as in the case of bending), causes an elastic behavior of the structure far above the limit assumed on the basis of a theory developed from the ideal plastic behavior of the material. The upper yield point is only partly influenced by the quality of the material and depends to a great extent on the velocity of the loading.[40] For slowly applied loads the ratio of upper and lower yield point will be greater than for rapidly applied loads. This is explained by the occurrence of the upper yield point, which may be considered as a "retardation of yield" similar to the phenomenon of the retardation of the boiling point of liquids. Instantaneously applied loads cause the upper yield point to drop.

Furthermore, it has been proved by tests that in many cases the maximum stresses in beams under loads have reached values considerably exceeding the yield point of the material established in the tension test. This "raised yield point" may amount to 1.2 to 1.7 of the yield point, in pure tension, depending on the cross-section of the beam. If the load is increased still further the cross-section reaches the plastic state instantaneously over its full height, and the stresses show a sharp drop from the upper yield point to the value of the lower yield point. Local instability (as, for example, buckling of the flanges, bending or torsional deformation) is often the result. The assumption of an

[40] "General Theory of Plasticity," by A. Freudenthal, Rept. I, 1, Second International Congress for Bridge and Structural Engineering, Berlin, Germany, 1936.

ideal plastic behavior of the material, as proposed by the author, therefore, has only a restricted application.

(3) On the other hand in the design of bridges fatigue is far more important than plasticity. The occurrence of pulsating and reversed stresses will always demand the design of bridges on the basis of the endurance limit of the material. Such a design will require a higher factor of safety than would be required by (plastic) limit design; therefore its use for such cases would always be imperative.

The endurance of a structure is particularly influenced by the form of the connections. It is diminished by notch action in gusset-plates as well as in welds, and by the thermal residual stresses in and around welds. One may conclude, therefore, that even under conditions favoring the application of the theory of limit design in the proposed form, the connections between the members will always represent weak and critical points where the validity of a design based on the ductility of the material alone may become questionable.

(4) The general conclusions to be reached on the basis of all the aforementioned facts are that the theory of limit design, in the form presented by the author, is applicable but to an extremely limited number of structures under permanent or slowly changing loads. It must not be applied to trusses even under fixed or slowly changing loads. In the design of bridges, where the endurance limit of the material, and the notch-action in the connections are the dominant factors, its application would not be justified.

Although the structural engineer is certain to be aware of the limits of the theory of elasticity, it does not seem justifiable to recommend its abandonment in favor of an unqualified acceptance of the theory of (plastic) limit design. It is certainly appropriate to endeavor to develop a theory which most closely fits the facts; but the theory of limit design, as presented by the author, can scarcely be regarded as fulfilling this requirement. Therefore, within certain limits, the theory of elasticity will remain the basic theory for the structural engineer, provided that the factor of safety is established on the basis of a thorough understanding of the actual behavior of the structure. This may involve the revision, and even the abandonment, of the current conception of stress.

The author is to be congratulated, sincerely, for having stressed this important fact.

HANS H. BLEICH,[41] ESQ. (by letter).[41a]—In his very instructive paper demonstrating the practical use of the theory of limit design the author extends its application to trusses. Such extension is not permissible because it is necessary that all compression members be so proportioned that the stresses induced will never reach the buckling limit. It is true that, if a compression member is loaded to its buckling capacity, a slight buckling will appear, the effect being to shorten the bar so that some other bar will begin to carry the load of the compression member. The extent of the buckling is limited,

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[41] Dr. Ing., Vienna, Germany.

[41a] Received by the Secretary April 18, 1939.

however, and must be very small. If the bar bends beyond a certain limit it can carry only a part of the full buckling load of the straight bar. An investigation of this permissible limit shows that, generally, compression members cannot be shortened as much as is required by the theory of limit design.

For simplicity, consider only the top panel of Fig. 15(*a*). According to the force diagram (Fig. 15(*f*)), the forces in Bars *c* and *d* are assumed to be

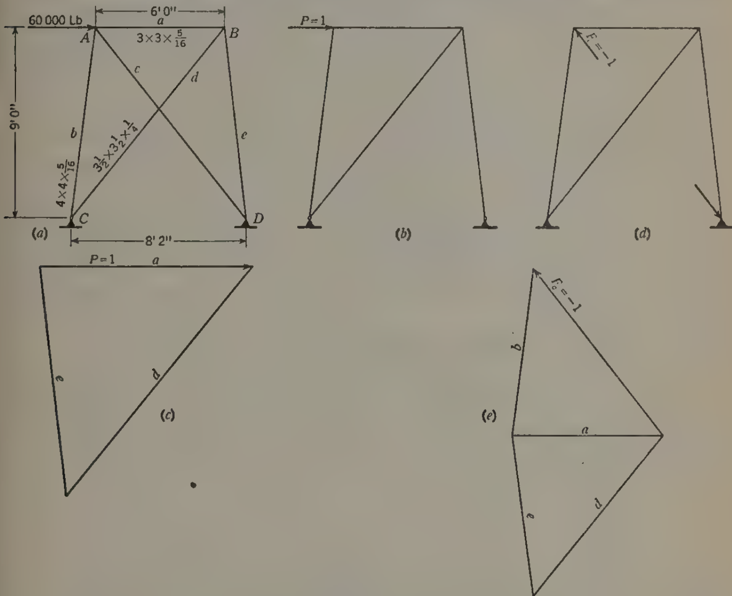

FIG. 34

35 000 lb and 50 000 lb, respectively. It is required to compute the shortening of Bar *c* in order to produce these total stresses in Bars *c* and *d*.

Fig. 34(*c*) and 34(*e*) are the force diagrams for this panel, subjected to the loads $P = 1$ and $F_c = -1$. In Table 2 the total stresses induced by Load $P = 1$ are F_a' and F_b'; and, the total stresses induced by Load $F_c = -1$ are F_a'' and F_b''. The change in the length A–D caused by the force $P = 1$ is $\Delta_P = \sum \frac{F\,F''\,l}{E\,A}$; and, the change produced by the force $F_c = -1$ is $\Delta_F = \sum \frac{(F'')^2\,l}{E\,A}$ (see Columns (7) and (8), Table 2). Thus, it is found that $\Delta_P = \frac{176.3}{E}$ and $\Delta_F = \frac{231.7}{E}$. If the change in the length of Bar *c*, caused by the buckling of this bar, is called Δ, the general equation for the force F_c is

$$P\,\Delta_P + F_c\,\Delta_c + \Delta = 0 \quad\quad (24)$$

TABLE 2.—COMPUTATION OF STRESSES IN THE TOP PANEL OF FIG. 15(a)

Bar (see Fig. 34(a))	Total stress, F', due to a load $P = 1$	Total stress, F'', due to a load $F_c = 1$	Length of member, L, in inches	Area, A, in square inches	$\frac{F'' L}{A}$	$F' \frac{F'' L}{A}$	$F'' \frac{F'' L}{A}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
a	−1.00	+0.70	72	1.78	+28.3	− 28.3	19.8
b	0	+0.79	109	2.40	+35.9	0	28.3
c	0	−1.00	137	1.69	−81.1	0	81.1
d	+1.40	−0.97	137	1.69	−78.6	−110.1	76.3
e	−1.10	+0.76	109	2.40	+34.5	− 37.9	26.2
Total	....	....	....	....	....	−176.3	231.7

Introducing the values of P, F_c, p, c, and E (= 30 000 000),

$$-\frac{176.3}{E}\,60\,000 + \frac{231.7}{E}\,35\,000 + \Delta = 0; \quad \text{or,} \quad \Delta = 0.083 \text{ in.}$$

Bar c (L = 137 in.) must buckle in two waves, as shown in Fig. 35, because Point E is fixed by the diagonal d. To find the deflections y_1 and y_2, the waves are assumed to be parabolic. The two parabolas must be similar so that $\frac{y_1}{y_2} = \frac{58}{79}$. The difference between the length of a parabola and the length of its chord is $\frac{8}{3}\frac{y^2}{L}$. The total change in the length of Bar c is:

$$\Delta = \frac{8\, y_1^2}{3 \times 58} + \frac{8\, y_2^2}{3 \times 79} = 0.0586\, y_2^2 \quad \ldots\ldots\ldots\ldots\ldots (25)$$

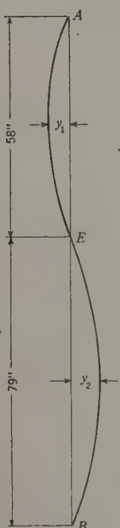

FIG. 35

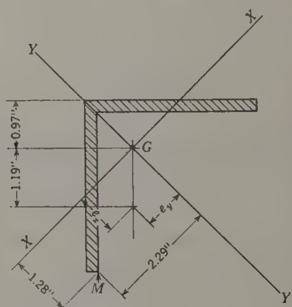

FIG. 36

The value of Δ necessary to produce the given stresses was found to be 0.083 in.; and the corresponding deflection was $y_2 = \sqrt{\frac{0.083}{0.0586}} = 1.19$ in.

Fig. 36 shows the cross-section of Bar c with its center of gravity, G, and the eccentricities, e_x and e_y, of the load with respect to the X and Y axes. The eccentricity is $e_x = e_y = 0.707\ y_2 = 0.83$ in.; the area of the cross-section is $A = 1.69$ sq in.; the moments of inertia are $I_x = 0.83$ in.^{4} and $I_y = 3.22$ in.^{4}; and, the greatest stress will occur in the corner of the angle, M:

$$s = \frac{35\,000}{1.69} + \frac{35\,000 \times 0.83}{0.83} 1.28 + \frac{35\,000 \times 0.83}{3.22} 2.29 = 86\,000 \text{ lb per sq in.}$$

This stress exceeds the elastic limit of 30 000 lb per sq in. The curved bar with a deflection of 1.19 in. is not able to carry the full buckling load. It follows that the entire system is unable to carry the load for which it was originally designed. The assumed redistribution of the load between Bars c and d can never occur because Bar c would have to buckle so much that it would lose about two-thirds of its buckling strength.

This example leads to the conclusion that either all compression members must be safe against buckling or, if a bar can be loaded beyond its buckling strength, the truss system without this critical bar must be able to carry the entire load (the small loading capacity of the buckled bar is neglected). Engineers made use of the latter possibility a long time ago in designing trusses with crossed diagonals in the center panels.

If the theory of limit design, limited by the foregoing statement, is used for the design of trusses, there will be no economical advantage over the theory of elasticity, except in a few special cases such as the aforementioned case of trusses with crossed diagonals. It was with this conviction that the writer stated,[9] in 1932, that the theory of limit design had better not be applied to trusses.

Alfred S. Niles,[42] Assoc. M. Am. Soc. C. E. (by letter).[42a]—The interesting question as to the proper method of determining the factor of safety of an engineering structure is reviewed by the publication of this paper. In practice the engineer has to choose between two classes of design methods. At present many structural engineers use what may be termed "working stress methods" in which they compute the stresses that would be developed by the maximum probable loads and so proportion the structure that these stresses do not exceed the "allowable working stresses" of the materials used. These allowable working stresses are obtained by dividing the pertinent "ultimate stresses" by the desired factor of safety. The alternative is to use what may be termed an "ultimate stress method" in which the maximum probable load is multiplied by the desired factor of safety to obtain the "ultimate load." The stresses that would be produced by this ultimate load are then computed and the structure so proportioned that these stresses do not exceed the appropriate

[9] *Bauingenieur,* 1932; Heft 19/20; p. 264.

[42] Prof., Aeronautic Eng., Leland Stanford Junior Univ., Aero Laboratory, Stanford Univ., Stanford University, Calif.

[42a] Received by the Secretary April 21, 1939.

ultimate stresses. The method of "limit design" described in the paper is obviously in the latter class. The author may protest against describing it as a method of comparing stresses, since he claims that emphasis should be placed on deformations rather than on stresses; but it is difficult for the writer to understand how the deformations can be determined without first computing the stresses that cause them.

The essential difference between the two basic design methods can be shown most clearly by the aid of a numerical example. Suppose it is desired to determine the adequacy of a 1-in. square bar of steel, 40 in. long, to carry simultaneously an axial compression of 6 250 lb and a uniformly distributed transverse load of 5.5 lb per in. It will be assumed that the bending moment at each end is zero, that Young's modulus is 30 000 000 lb per sq in., and that the compression yield point of the material, 36 000 lb per sq in., is the appropriate ultimate stress. For the 1-in. square cross-section the moment of inertia, I, is $\frac{1}{12}$ and the section modulus, $\frac{I}{y}$, is $\frac{1}{6}$ · The formula[43] for the maximum bending moment resulting from the assumed loading is

$$M_{\max} = w\,j^2 \left[1 - \sec\left(\frac{L}{2\,j}\right)\right] \qquad (26)$$

in which: w is the transverse load; L is the length of the member, $j = \sqrt{\frac{E\,I}{P}}$; E is Young's modulus; I is the moment of inertia of the cross-section; and, P is the axial compression.

If the desired factor of safety is 2.0 and the working stress method is used, the maximum permissible value for the total unit stress, $f = \frac{P}{A} + \frac{M\,y}{I}$, will be $\frac{36\,000}{2} = 18\,000$ lb per sq in. Using Equation (26) to determine the imposed stress, $j^2 = \frac{E\,I}{P} = \frac{30\,000\,000}{12 \times 6\,250} = 400$; $j = 20$ in.; $\frac{L}{2\,j} = 1.00$; $\cos\left(\frac{L}{2\,j}\right) = 0.5403$; and, $1 - \sec\left(\frac{L}{2\,j}\right) = 0.8508$. Then $M_{\max} = 5.5 \times 400 \times 0.8508 = 1\,872$ in-lb. The total imposed stress is therefore $f = \frac{P}{A} + \frac{M\,y}{I} = \frac{6\,250}{1} + 1\,872 \times 6 = 17\,482$ lb per sq in. Thus, according to this method of computation the member in question would be adequate, the desired factor of safety having been provided with a margin of about 3 per cent.

Suppose, however, the investigation were made by the ultimate stress method. The analysis would then be made for an axial load of 12 500 lb acting in conjunction with a transverse load of 11.0 lb per in., whereas the allowable total stress would be 36 000 lb per sq in. From these data, $j^2 = 200$; $j = 14.14$ in.; $\cos\left(\frac{L}{2\,j}\right) = 0.15615$; and, $1 - \sec\left(\frac{L}{2\,j}\right) = 5.4041$. Then $M_{\max}$

[43] "Airplane Structures," by Alfred S. Niles and Joseph S. Newell, John Wiley & Sons, Inc., New York, N. Y., 1938, Vol. II, Second Edition, p. 98.

= 11 889 in-lb, and the maximum unit stress becomes f = 83 834 lb per sq in., which is more than twice the allowable value of 36 000 lb per sq in.

This example is an extreme one, but it illustrates the difference between the two basic methods and shows that the decision as to whether to use a working stress or an ultimate stress method of analysis may have a great influence on the adopted design sizes. Using the working stress method the 1-in. square rod was found adequate by a small margin; using the ultimate stress method, however, it was found inadequate by a large margin.

The ultimate stresses that may be used in either type of procedure are of various kinds. Some, like the ultimate tensile strength, are the maximum true stresses that can be resisted without causing fracture of the material. Others, like the yield point, are stresses at which plastic flow and permanent set begin to be excessive. Still others, like the stresses used in column design, are those at which a member becomes unstable, due either to elastic deformation, plastic flow, or a combination of the two. Finally, there are ultimate stresses in common use, like the modulus of rupture, which are not true stresses but merely convenient numbers which represent, indirectly, the maximum load that a member can carry.

Before an intelligent selection can be made of a general method of design and the ultimate stresses to be utilized, it is necessary to consider in more detail what is to be recognized as "failure." The most serious type of failure is that which occurs when a structure becomes unstable and collapses. Simple examples of this are failures of a truss due to the elastic buckling of a long slender strut or to a tension member being pulled away from its anchorage. In some structures, however, instability and collapse may be preceded by excessive deformations due to plastic flow, and such conditions are properly to be regarded as constituting a type of failure. The elongation of a bolt hole due to over-load is an example of this type, and may be termed "plastic failure" to distinguish it from the previously mentioned "instability failure."

In order to obtain the most economical designs consistent with safety, both types of failure must be considered, and it is advisable to use different ultimate stresses and factors of safety for the two types. This has been the normal practice in airplane design for several years, and it has proved to be quite satisfactory.

The factor of safety that should be provided for any specific part of a structure depends on both the type of failure it is intended to prevent and the type of ultimate stress used. For plastic failure, the yield point is the most appropriate ultimate stress, and since experience has indicated that a factor of safety of 1.00 is sufficient, that is the value specified by the Civil Aeronautics Authority. This requirement is governing only where it would take a much larger load to cause collapse than that needed to produce excessive plastic flow. It is also equivalent to the author's proposal that a design be checked to insure that the maximum probable load will not cause stresses in excess of the yield point. The justification for such a low factor of safety against plastic failure is that, even if the assumed maximum probable load should be exceeded

in service, the result would not be disastrous but would merely cause some deformations of the structure which normally could be repaired satisfactorily.

The factor of safety to be provided against instability failure, however, should exceed 1.00 since otherwise a load in excess of the assumed maximum that is probable would be likely to cause a disastrous collapse. The desirable magnitude of this factor of safety depends on many influences, including the importance of weight economy, the reliability and precision of the methods of analysis, the uniformity of materials used, and the quality of workmanship to be expected. In aeronautical work the importance of weight economy is so great that a factor of 1.50 is the usual value, but larger values are specified for members and types of construction for which experience has shown it to be difficult to predict the strength with precision.

For the benefit of the engineer who may consider these aeronautical factors of safety to be dangerously low, the writer believes a few explanatory remarks are in order. It must first be remembered that these factors are not applied to the loads encountered in normal flying, but only to the most severe loads likely to be encountered in strenuous maneuvers or very rough air. The factors provided against the loads encountered in normal flight are several times the foregoing values. Secondly, the maximum load to which an airplane can be subjected in flight can be predicted with more confidence than the weight of the heaviest truck that may try to cross a new highway bridge. In the third place, airplane designers have the benefit of many static tests to destruction to guide them, but full-scale destruction tests of bridges and buildings are most unusual. As a result, the airplane designer can be more confident that he actually has provided a factor of safety of 1.5 than many designers of other structures can be that they have provided their nominal 2.0 or more. Finally, the fact should be mentioned that major structural failures of airplanes are very rare, and aeronautical engineers therefore consider that, although the factors of safety they use are very low, they are adequate if proper care and skill are used in developing their designs.

The factor of safety may be defined as either the ratio of the ultimate stress to the stress produced by the maximum probable load, or as the ratio of the load that would cause failure to the maximum probable load. The use of a working-stress method of design implies the use of the former definition, whereas the use of an ultimate-stress method implies the latter. If internal stresses remained strictly proportional to external loads right up to the point of failure, it would make no difference which definition and method of design were used. In actual structures, however, this proportionality up to failure is the exception rather than the rule, and usually the second definition results in a lower value for the factor of safety for a given design than the former.

A common type of structural member in which the internal stresses are not directly proportional to the external loads is the "beam-column," or member subjected to combined axial compression and transverse bending loads. If such a member is subjected to a loading that produces a maximum internal stress equal to half the ultimate, the external loads can be increased until the maximum unit stress is doubled before failure takes place. This result will

be obtained, however, before the external loads have been doubled. Thus, when the factor of safety in terms of unit stresses is 2.00, it would be less than 2.00 in terms of external loads. In situations of this character it appears much more reasonable to define the factor of safety in terms of load than in terms of stress. A logical consequence is that an ultimate-stress method rather than a working-stress method should be used in design.

An additional reason for this practice is that the use of an ultimate-stress method permits a designer to check the reliability of his methods of analysis by static tests to destruction. If a structure is designed to a factor of safety of 1.50 by an ultimate-stress method, the static test will show, clearly and directly, whether or not the desired end has been achieved. However, if it is designed to a factor of safety of 1.50 by a working-stress method, it is practically impossible to use destruction tests to determine whether the desired end has been achieved. In such a case it may be that the tension members should not fail until the structure is subjected to 1.50 times the design load, whereas some compression members may be expected to fail under 1.35 times the design load. If the compression members failed under 1.38 times the design load, their adequacy would have been established, but the adequacy of the tension members would remain undetermined. Any attempt to check designs made by a working-stress method by means of static tests to destruction involves a mass of interpretative computations which are of doubtful validity. Thus, if tests are to be made (and they are clearly desirable if economically feasible), the designs must be made by an ultimate-stress method.

In much structural design the deviation of the internal stresses from strict proportionality to the external loads is small, the factors of safety provided are relatively large, and weight saving is of minor importance. As a result, working-stress methods of design have given satisfactory results. In airplane design these conditions have not obtained. Types of construction are in common use in which the deviations of the internal stresses from strict proportionality to external loads are large, factors of safety are relatively small, and weight saving is of the utmost importance. As a consequence of these conditions the airplane designers have, for a long time, been accustomed to using ultimate-stress methods of design.

Some types of structural members, notably struts of moderate slenderness, become unstable as soon as the yield point stress is reached, or the load that can be added after the yield point has been reached is negligible. For the design of such members against instability failure the yield point of the material is a logical ultimate stress to be used in practice. Other types of members, such as rectangular beams, must be subjected to considerable increments of load after the yield point has been reached before they collapse. The writer is in complete accord with the author in protesting against the use of the yield point as the ultimate stress in the design of such parts.

In designing to obtain a desired factor of safety against collapse, particularly when it is desired to take advantage of any capacity of the structure to carry a larger load than the minimum necessary to produce elastic buckling or plastic flow in some of its elements, the methods used to compute the imposed

stresses must be considered in the selection of the corresponding ultimate stresses. The most common practice is to compute the stresses that would be caused by the ultimate design load under the assumption that the stress distribution would be the same as if the material remained elastic, even though it is recognized that the resulting computed stresses are fictitious. This is the sensible thing to do whenever the corresponding ultimate stresses can be suitably determined. An example is a beam subjected to simple bending. At maximum load the actual maximum unit stress in such a beam is difficult to determine, but it is relatively easy to determine a "modulus of rupture," assuming that the formula $f = \frac{M\,y}{I}$ remains valid.

With some other types of structure this procedure is inapplicable, as some parts will buckle elastically under a fraction of the load needed to cause failure of the whole. In most cases of elastic buckling of a redundant member, the buckled member continues to carry the load which caused the buckling but refuses to carry any more when additional external load is imposed on the structure. Thus, when a plate girder with a very thin web is subjected to simple bending under low loads, the shear on the girder is transmitted primarily by shear stresses in the web. These shear stresses may be resolved into diagonal tensile and compressive stresses acting at right angles to each other. As the external load is increased, the diagonal compressions cause the web to buckle into a series of waves or wrinkles parallel to the diagonal tensile stresses. If the web were not provided with suitable vertical stiffeners, this buckling would result in instability and collapse of the entire beam. When the stiffeners are provided, however, increases in shear can be carried by increased diagonal tensions in the web, compression in the vertical stiffeners, and suitable stresses in the chord members, and that is just what takes place. If the buckling of the web into wrinkles were considered to represent failure, the ultimate strength of the girder would often be only a small fraction of the load actually required to cause collapse. The airplane designer recognizes this fact and uses the actual load-carrying capacity of the girder as its ultimate strength for use in design. In computing the stresses present when the girder becomes unstable the more conservative method is to disregard the diagonal compressive stresses that are actually carried across the wrinkles. Some airplane designers, however, prefer to include these compressive stresses as part of the resistance of the girder to load, and to count on this resistance in design.

A similar situation is found in the panels of thin sheet metal with longitudinal stiffeners that are frequently used in the construction of airplane fuselages. In this construction the parts of the sheet midway between stiffeners buckle elastically under relatively low loads. As the loads increase, the width of the buckled sheet also increases, increments of load being carried by the stiffeners and the parts of sheet adjacent to them. Eventually only the stiffeners are able to provide resistance to increments of load, and when the critical stresses are reached in the stiffeners, the entire structure collapses. One might say that the proper method of design would be to consider only the strength of those parts of the combination that are still accepting increases in load when

the ultimate load is approached in computing the strength of the panel. This is not the practice, however, as the parts that have buckled elastically are still carrying some of the load, and the carrying capacity of those parts is included in computing the strength of the unit. The magnitude of this supporting effect of elements that will have buckled elastically is often difficult to predict, and the question is the subject of much research, both theorctical and experimental.

Although it is common practice in airplane design to take cognizance of the changes in stress distribution due to elastic buckling in computing ultimate loads, it is not usual to do the same with respect to changes in stress distribution caused by plastic flow. There would appear to be no valid objection to doing this if the detailed procedure employed were logically sound. The author offers certain suggestions along this line, but some of his detailed proposals are open to serious question.

In his first case the author assumes that after the elastic-limit stress has been reached at the supports of a beam fixed at both ends, the section at each support acts as a pin joint. Thus, if the load of 10 000 lb per ft produces a moment of 30 000 ft-lb and that moment produces the yield-point stress in the outer fibers, the resisting moment at the support under a load of 13 333 lb per ft will also be only 30 000 ft-lb. This is far from being true. Under 10 000 lb per ft there will be linear variation of stress over the cross-section, and only the outermost fibers will be subjected to the yield point stress. Under any additional load the deformation of the end sections will increase, more and more fibers will be subjected to the yield point stress, and the resisting moment will increase. The result will be that the movement of the points of inflection toward the supports will be less than that estimated by the author. From this it might be argued that the author's method is conservative, and that may be true, but before applying it in practice a more careful study of all the factors involved should be made.

In his second to fourth cases the author makes a study of the effect of unloading and repetition of load, but omits any consideration of the distribution over the cross-section of the residual strains and internal stresses that he assumes to exist. His conclusions may be useful in stimulating research but should not be used as a basis of design until the distribution of such strains and stresses has been more thoroughly investigated.

Another factor that should be given careful study before adopting the author's detailed procedure for cases involving plastic flow is the shape of the stress-strain diagram of the material used. Ordinary structural steel has a definite yield point, but this is not true of structural aluminum alloys, stainless steel, or many of the high-strength steels that are coming into use. This means that an important factor underlying the author's proposals is lacking when applied to the materials used where economy of weight is an important consideration. His theory may be modified to apply to these materials without definite yield points, but for these, as well as for structural steel, it requires more extensive digestion than is indicated in the paper.

A. FLORIS,[44] ESQ. (by letter).[44a]—When the stresses in some parts of the statically indeterminate structure reach the plastic limit of the material, other less stressed parts come into play, thus causing a better distribution of stress. This equalization, commonly called "self help" of the redundant structure, is the thesis of the author's paper on limit design.

In some instances, this equalization occurs quite naturally, without the aid of the plastic property of the material. In a beam fixed at both ends and carrying a vertical concentrated load in the middle of the span, for example, the bending moments at both ends and in the middle are equal and of opposite signs. Within the elastic limit of the material, therefore, the ratio of positive and negative moments is 1 : 1. Such cases are rare, however, and, in general, recourse must be taken to the plastic behavior of the material, if an equalization of stress is to be achieved.

In this thought provoking paper the author treats the fixed-end beam exclusively. Unfortunately, the use of this structural element is rather restricted in practice. Continuous structures—beams (which are very common) as well as rigid frames—have been ignored. No doubt, this is a very serious omission.

Steel structures of these types can be analyzed conveniently by means of factors that take care of the moment reduction caused by the plastic flow of the material. As soon as this stage is reached, imperfect hinges are formed in places where the bending moments have their greatest value, thus relieving the strains by redistribution. The ensuing greater freedom of motion in the structure can easily be counterbalanced by the remaining numerous constraints of the manifold redundant structures.[45] In the light of the theory of plasticity, therefore, highly redundant systems are safer against the effect of unforeseen emergencies, including settlements of supports and destructive earthquakes. This is quite the opposite of the results obtained by the elastic theory.

Although the theory of plasticity is readily applicable to steel structures, in the case of reinforced concrete the problem is more complicated. In this composite material, only the steel reinforcement has a well-defined yield point; the concrete is a brittle material. However, as G. von Kazinczy has shown experimentally,[46] the compressive strength of concrete remains constant during the initial yielding of steel. On the basis of this fact it is possible to develop a theory of stress analysis and design.

In statically determinate framed structures it is always possible to stress all bars to the limit; but it is never possible in internally redundant frames, if the dimensioning is done in accordance with the elastic theory. In such structures of n-fold static indeterminacy, it is possible to stress all except n bars to the limit. These n bars constitute, so to speak, the reserve strength of the frame. Consequently, the greater the number of redundants in relation

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[44a] Received by the Secretary May 23, 1939.

[45] "Der Momentenausgleich durlaufender Traggebilde im Stahlbau," by Felix Kann, Berlin, Germany, 1932.

[46] "Die Plastizität des Eisenbetons," by G. von Kazinczy, *Beton u. Eisen*, 1933, p. 74.

to the remaining bars, the greater will be the economic advantages by applying the theory of plasticity.[47]

Framed structures designed either by the elastic or plastic theory are perfectly safe. The advantages gained by the use of the theory of plasticity are economy of material and simplicity of analysis. Thus, the most economical solution can be found with the least effort.

Redundant compression bars cannot be included in the analysis of the structure for the following reasons: When the buckling strength is reached, the resistance of the bar ceases and the supporting capacity of the redundant frame is reached almost instantaneously. This has been demonstrated by several tests.[48] The factor of safety is much larger for compression than for tension bars. The latter can sustain considerably plastic deformations without greatly endangering the safety of the structure. However, as soon as the former reach their buckling strength, the frame will collapse immediately. The redundant tension bars having a smaller factor of safety will reach the plastic stage earlier than the redundant compression bars with a greater factor of safety.

For example, by removing the redundant bar of a frame, statically indeterminate in the first degree, in which the tension reaches the permissible stress first, a force is applied in the direction of this bar equal to the allowable unit tensile stress multiplied by an arbitrarily chosen cross-section. This force, combined with the external loads, determines the axial forces of the bars, which later are dimensioned on the basis of the allowable unit stress. In designing a framed structure in accordance with the theory of plasticity, therefore, the permissible unit stress is not eliminated. The method proposed in this paper, to design framed structures for an over-load capacity in order to avoid the use of the allowable unit stresses, is objectionable because of the foregoing reason concerning factors of safety.

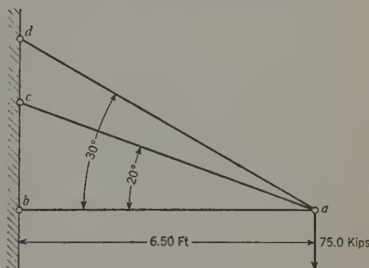

FIG. 37.—CANTILEVER FRAME

In the following a simple example will illustrate the points in question: The cantilever frame composed of Bars *ab*, *ac*, and *ad*, Fig. 37 has a span of 6.5 ft, carries a load of 75.0 kips and is statically indeterminate in the first degree. All double angles are $\frac{3}{8}$ in., back to back, and all bars have welded connections.

By using the elastic theory, the axial force in any of the three bars can be chosen as the redundant quantity. In the present case the axial force of Bar *ad* is taken as the redundant. Inasmuch as the frame has one degree of statical indeterminacy, the stresses in one bar only cannot be utilized completely. This is Bar *ac*, as the results of the elastic theory have shown. After repeated trials, the most economical solution

[47] "Zuschrift," by E. Melan, *Der Bauingenieur*, 1938, p. 488.

[48] "Drei Beiträge zur Frage des Tragvermögens statisch unbestimmter Stabwerke," by . Chwalla, International Assoc. for Bridge and Structural Eng., 1933–1934, Vol. II, p. 6; see also "Versuche mit innerlich statisch unbestimmten Fachwerken," by G. von azinczy, *Der Bauingenieur*, 1938, p. 236.

has been determined, as given in Table 3, in which the solution of the statically determinate frame is included for the purpose of comparison.

According to the results of the elastic theory, Bar *ad* is the only tension member in the frame. Even in a different combination of sections and possible

TABLE 3.—VARIOUS SOLUTIONS OF THE CANTILEVER FRAME IN FIGURE 37

Bar	STATICS			ELASTICITY			PLASTICITY		
	Dimensions, in inches	Axial force, in kips*	Unit stress, in kips per square inch*	Dimensions, in inches	Axial force, in kips*	Unit stress, in kips per square inch*	Dimensions, in inches	Axial force, in kips*	Unit stress, in kips per square inch*
ad	....	....	....	2—4 × 3½ × ⅝	+154.0	+17.9	2—4 × 4 × ½	+135.0	+18.0
ac	2—7 × 3½ × ⅝	+219.0	+17.8	2—2½ × 2½ × ¼	− 8.0	− 3.4	2½ × ½	+ 22.0	+17.6
ab	2—6 × 6 × ⅝	−206.0	−14.5	2—4 × 4 × ⅝	−125.0	−13.6	2—4 × 4 × ⅝	−137.0	−14.8

* Plus denotes tension and minus denotes compression.

reversal of sign in Bar *ac*, the tension in Bar *ad* will always reach the permissible stress first. In all cases Bar *ab* is under compression and consequently is excluded from consideration. For this reason only Bar *ad* can be subjected to plastic deformation.

By removing and replacing Bar *ad* by a tensile force, equal to the product of allowable tensile stress by an arbitrarily chosen sectional area, the axial forces in the other bars are determined by statics. Selecting for Bar *ad* a smaller section than that given by the elastic theory, it is obvious that the stress in this bar will be nearer to the plastic deformation than in all other bars. However, inasmuch as these bars, in accordance with the design, possess a greater reserve strength, it is obvious that the force distribution or equalization of stress will take place below the limit of safety. The data of the frame analysis based on the foregoing theory are also given in Table 3.

Comparing the statically determinate frame with the redundant frame, the savings are: For the elastic theory, 20.3%, and for the theory of plasticity, 29.5 per cent. On the other hand, comparing the results of the theory of plasticity with those of the elastic theory, the saving is 11.5 per cent.

The theory of plasticity can be applied also to externally redundant framed structures. Tests on a continuous, framed beam have demonstrated that this is permissible.[49]

[49] "Tragfähigkeitsversuche an einem durchlaufenden Fachwerkbalken aus Stahl," by G. Grüning and E. Kohl, *Der Bauingenieur*, 1933, p. 67.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

SPECIFICATION AND DESIGN OF STEEL GUSSET-PLATES

Discussion

By C. W. Wixom, Assoc. M. Am. Soc. C. E.

C. W. Wixom,[21] Assoc. M. Am. Soc. C. E. (by letter).[21a]—A subject on which little information has been available is presented in this paper. Mr. Rust recommends that the non-uniformity of stress among rivets be minimized by making the groups short and compact without elaborate attempts to increase net sections. This recommendation has been strikingly supported in the paper entitled "Tension Tests of Large Riveted Joints."[22] These latter tests, however, led to the further conclusion that the present practice of assuming equal rivet stresses is satisfactory. Consistently, this assumption should be made not only in determining the number of rivets but when investigating gusset-plate stresses.

Tests and analyses of splices in which the stresses are principally direct and central, although valuable in themselves, are of limited application in analyzing gusset-plates in which stresses are transferred principally by shear and flexure. However, in the case where a gusset forms part of a chord splice, there is a large direct stress and also considerable bending stress in the gusset due to the location of the chord near the edge of the gusset. It is sometimes desirable to separate the splice from the gusset and eliminate this source of bending.

The statement that "An ideal gusset approaches a circle in shape, or a polygon of approximately equal axes, with all the forces intersecting at or near its center" will not be accepted by all (see heading "Recommendations for the Specifications and Design of Steel Gusset-Plates: General Practice"). The usual gusset connects one or two diagonals (Fig. 20 or 21), a vertical, and a chord. The "ideal" shape of such a gusset just encloses the rivet groups and is rectangular. The point at which the forces intersect is seldom near the center of the gusset. The gusset in Fig. 13 has been superimposed in

Note.—This paper by T. H. Rust, Assoc. M. Am. Soc. C. E., was published in November, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1939, by Messrs. R. H. Sherlock, and L. E. Grinter; and April, 1939, by Russell C. Brinker, Jun. Am. Soc. C. E.

[21] Asst. Engr., Am. Bridge Co., New York, N. Y.

[21a] Received by the Secretary May 9, 1939.

[22] *Proceedings*, Am. Soc. C. E., May, 1939, p. 805.

dashed lines for comparison on Fig. 20. The entire stress from the vertica is transferred as a shear between the vertical and the diagonal. An extensio of the gusset to the left of the vertical is of no assistance in this shear transfe An experienced detailer can frequently lay out an efficient gusset with

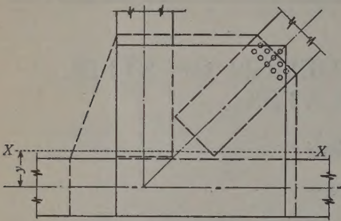

Fig. 20

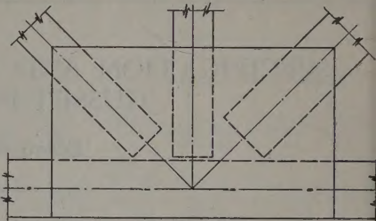

Fig. 21

rectangular corner at the diagonal. The stress in the corner rivets will compar favorably with the others because the plate is converging with the numbe of rivets.[23, 24] Lug angles, or clip angles, have been shown to be flexible an inefficient.[25, 26] They add to the complexity of the stress path.

Since the lines of force intersect at a point, difficulty is sometimes experi enced in visualizing bending in gussets. Perhaps the simplest illustration i that of section "*X–X*," Fig. 20, on which there is a bending moment equal t the increment of chord stress times its lever arm "*y*." The author has quote the method of analyzing more complex sections.[15]

Gusset-plates should be designed with adequate proportions without undu concern, except in extreme cases, about fabricating limitations or secondar stresses. Many large gussets have been carefully and successfully fabricate for modern structures. Secondary stresses resulting from truss rigidity ma be eliminated to a large degree by proper cambering—that is, by fabricatin with lengths of members, but not the angles between the members, correcte for their deformation under dead load plus a percentage of live load. In thi manner, reverse secondary stresses are induced during erection and these ar relieved when the loads are applied.

Of two structures designed for the same duty, the one having the fewe members will often be the more economical—not only in first cost but i maintenance. It will be easier to paint. Since its members and gussets ar larger, it will vibrate less and develop fewer loose rivets. Painting of contac surfaces not only reduces the friction but results in loose rivets.[27] It is no prohibited in the leading specifications.[28, 29, 30]

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[23] "Modern Framed Structures," by the late J. B. Johnson, the late C. W. Bryan, M Am. Soc. C. E., and F. E. Turneaure, Hon. M. Am. Soc. C. E., Part III, p. 112.

[24] "Riveted and Bolted Connections," by C. Batho, First Rept. of Steel Structure Research Committee, pp. 115–120.

[25] "Modern Framed Structures," by the late J. B. Johnson, the late C. W. Bryan, M Am. Soc. C. E., and F. E. Turneaure, Hon. M. Am. Soc. C. E., Part III, p. 127.

[26] "Riveted and Bolted Connections," by C. Batho, First Rept. of Steel Structure Research Committee, p. 102.

[15] "Bridge Engineering," by the late J. A. L. Waddell, 1925, pp. 519–529.

[27] "Tests on Effect of Paint between Riveted Plates," by the late Richard Khuen, Jr. M. Am. Soc. C. E., *Engineering News-Record*, Vol. 90, 1923, p. 460.

[28] Am. Ry. Engineering Assoc. for Steel Railway Bridges, 1928, par. 544.

[29] Am. Assoc. of State Highway Officials for Highway Bridges, 1935, par. 3, 11, and 9

[30] Am. Inst. of Steel Construction for Structural Steel for Buildings, 1937, par. 26(c)

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DISCUSSIONS

EARTHQUAKES AND STRUCTURES

Discussion

By A. A. Eremin, Assoc. M. Am. Soc. C. E.

A. A. Eremin,[28] Assoc. M. Am. Soc. C. E. (by letter).[28a]—The moment of inertia of mass has a negligible effect on the distribution of earthquake stresses in the vertical long (compared with transverse dimension) beam, as correctly stated in the paper. Likewise, one may easily find that the error from neglecting the influence of moment of inertia of mass in a vertical short beam is less than that from neglecting the influence of shear stresses. If the moment of inertia of mass is not considered a more direct development of the equations may be made.

Equation (7) may be derived from Equation (1), and from the well known elastic equation: which is,

$$E\,I\,\frac{\partial^4 y}{\partial x^4} = -\frac{\partial V}{\partial x} \qquad (54)$$

Likewise, Equation (26) may be developed from Equation (1) and from the elastic equation:

$$E\,I\,\frac{\partial^4 y}{\partial x^4} = -\frac{\partial V}{\partial x} + P\,\frac{\partial^2 y}{\partial x^2} \qquad (55)$$

Note.—This paper by Leander M. Hoskins, Esq., and John D. Galloway, M. Am. Soc. C. E., was published in December, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1939, by Messrs. Homer M. Hadley, and R. McC. Beanfield; April, 1939, by Messrs. R. S. Chew, Jacob J. Creskoff, and Arthur C. Ruge; and May, 1939, by Walter L. Huber, M. Am. Soc. C. E.

[28] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[28a] Received by the Secretary April 12, 1939.